

Disclaimer: This list is meant to be an *approximate* guide; please note that you are responsible for all the material covered in the course.

General Rules:

- **Bring several sharp pencils and an eraser. Calculators will not be allowed on the exam.**
- Brush up on basic tools from pre-calculus! Some of them are listed here. More can be found in the front of the book (the first page, right before the title page).

1. Laws of exponents:

$$a^m a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad a^{-n} = \frac{1}{a^n}, \quad (a^m)^n = a^{mn}$$

2. Factoring the **difference** of **like** powers:

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + ab^{n-2} + b^{n-1})$$

3. Expanding:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

4. If $P = (x, y)$ is a point on the unit circle centered at the origin, then $x = \cos \theta$ and $y = \sin \theta$, where θ is the angle made with the x -axis by the line joining the origin and P . This immediately allows you to easily deduce the sines and cosines of angles like $0, \pm\pi/2, \pm\pi, \pm3\pi/2, \dots$

- Start by reviewing past quizzes. Solutions were put up on the board, and discussed in class carefully. Your notes should contain these solutions. Don't just read the solutions, work them out again from scratch and check your work! Modify the problems. See if you can still solve them.
- Do the same with problems that were solved in class. Now repeat the process with the assigned homework problems.

The questions below test how well you have *understood* the underlying *concepts*.

1. Explain clearly in words what the symbols $\lim_{x \rightarrow c} f(x) = L$ mean. Then give a precise mathematical definition.
2. If $\lim_{x \rightarrow \infty} f(x) = L$, what does this tell us about the graph of f ?
3. When we say that the line $y = 5$ is a horizontal asymptote of the graph of the function $y = f(x)$, what do we mean? Explain clearly in words, and then use symbols.
4. When we say that the line $x = 5$ is a vertical asymptote of the graph of the function $y = f(x)$, what do we mean? Explain clearly in words, and then use symbols.

5. Know the conditions under which limit laws can be applied. For example, what conditions are needed for $\lim_{x \rightarrow c} (f(x)g(x)) = \lim_{x \rightarrow c} f(x) \lim_{x \rightarrow c} g(x)$ to be true?

6. What is the Sandwich Theorem, and when is it useful?

7. What are the values of

- $\lim_{x \rightarrow 0^+} \frac{1}{x}, \quad \lim_{x \rightarrow 0^-} \frac{1}{x}$
- $\lim_{x \rightarrow \infty} \frac{1}{x}, \quad \lim_{x \rightarrow -\infty} \frac{1}{x}$.
- $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}, \quad \lim_{x \rightarrow \infty} \frac{\sin(x)}{x}$
- $\lim_{x \rightarrow 0} \frac{\cos(x)}{x}, \quad \lim_{x \rightarrow \infty} \frac{\cos(x)}{x}$

Understand why these values are what they are. Don't memorize! Learn to **reason** it out.

8. When is a function continuous at an interior point in its domain?

9. If f is continuous at x_0 , and $\lim_{x \rightarrow x_0} f(x) = L$, what can be said about the value of $f(x_0)$?

10. Know examples of functions that are continuous.

11. Know examples of functions that are discontinuous.

12. What are the ways in which we can combine continuous functions to obtain new continuous functions?

13. What does the intermediate value theorem say? What are the hypotheses? What is the conclusion? Why do we need the hypothesis of continuity? (See p.55, Fig. 1.69 and accompanying explanation.)

14. Explain how the intermediate value theorem can be used to deduce existence of solutions to equations.

15. Clearly describe the process of using secant lines to construct approximations of the tangent line to a curve at a point.

16. Distinguish between the *average* rate of change of a function over an interval and the *instantaneous* rate of change of a function at a point.

17. What is the precise definition of the slope of the tangent line at a point on a curve? (see p.73)

18. What do we mean by the **derivative of a function f at a point x_0** ? How do we denote this quantity? What is its connection to the average rate of change of f over an interval containing x_0 ? What is its connection to the instantaneous rate of change of f at x_0 ?

19. Give four interpretations of the difference quotient (see p. 73)

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

20. There are several notations used for the derivative. Be proficient at using all of them.

21. If f is continuous at x_0 , must f be differentiable at x_0 ? (Consider $f(x) = |x|$.)
22. Given the graph of a function, know how to graph its derivative.
23. Application of the above: given the graph of position against time, make deductions about the velocity function.
24. Another application, in reverse: Give the graph of the velocity of an object, make deductions about its trajectory (Problem 15, p.100).
25. Know how to calculate the derivative of a given function f using the **definition**. (p. 77, Solved Example 2, p. 82, exercises 1, 3)
26. Know the rules of differentiation in 2.3 and 2.5, and how to calculate derivatives using these rules.