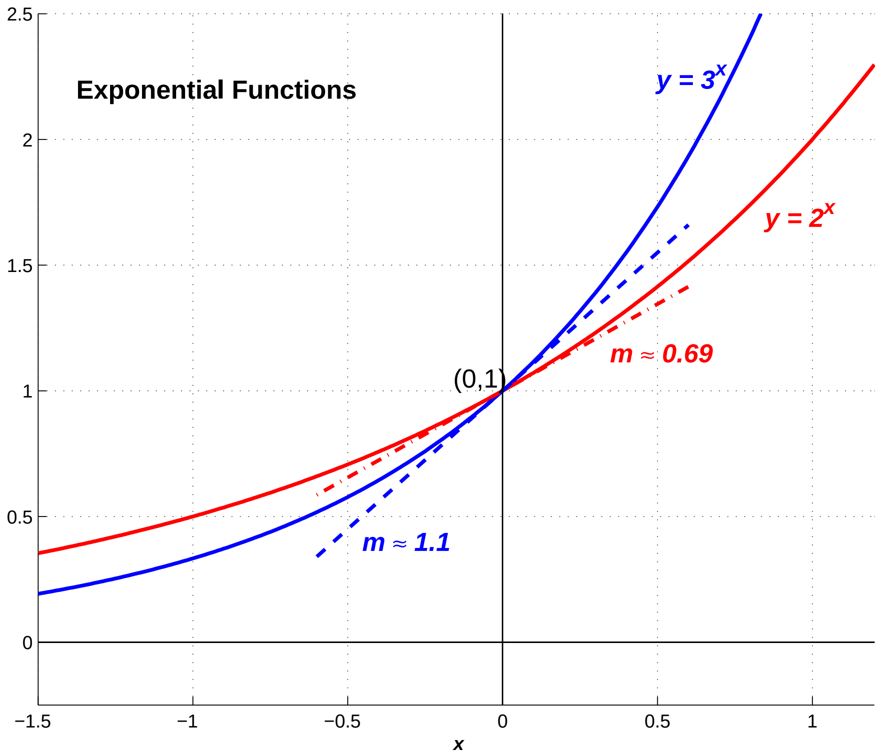
Consider the exponential curves $y = a^x$, for different values of the base a. Increase the value of the base from $a = 2$ to $a = 3$.



In each case, we examine the slope of the tangent line to these curves at $x = 0$. These tangent lines all go through the point $(0, 1)$, but with different slopes. When $a = 2$, the slope is approximately

$$\frac{2^{0.0001} - 2^0}{.0001 - 0} = \frac{2^{0.0001} - 1}{.0001} \approx .69$$

When $a = 3$, the slope is approximately

check on your calculator!

$$\frac{3^{0.0001} - 3^0}{.0001 - 0} = \frac{3^{0.0001} - 1}{.0001} \approx 1.1$$

Thus, as the value of the base a continuously increases from 2 *to* 3*, the value of the slope at* $x = 0$ *increases continuously from about .69 to around 1.1*

By the intermediate value theorem, there must therefore be a value of the *base* between 2 and 3, such that the slope at $x = 0$ of the corresponding exponential curve is *exactly* equal to 1.

This special value of the base is denoted by e. The value of e lies between 2 and 3. It can be shown that e is not a rational number and its value to 13 decimal places is $2.7\,1828\,1828\,4590\ldots$

When we speak of *the exponential function* we mean the function $f(x) = e^x$. It has the special property that its slope at $x = 0$ is exactly 1.