1. Investigate the area under $f(x) = 1/x$ from $x = 1$ to $x = \infty$.

$$\int_1^\infty \frac{1}{x}\,dx$$

(a) Find an antiderivative first: $\int \frac{1}{x}\,dx =$

(b) $\int_1^{10} \frac{1}{x}\,dx =$

(c) $\int_1^{100} \frac{1}{x}\,dx =$

(d) $\int_1^{1000} \frac{1}{x}\,dx =$

(e) Do we see a pattern? What is it? $\int_1^{} \frac{1}{x}\,dx =$

(f) We expect $\lim_{a\to\infty} \int_1^{a} \frac{1}{x}\,dx =$

(g) Prove it!

i. First, $\int_1^{a} \frac{1}{x}\,dx =$

ii. Hence $\lim_{a\to\infty} \int_1^{a} \frac{1}{x}\,dx = \lim_{a\to\infty} \qquad =$

2. Investigate the area under $f(x) = 1/x^2$ from $x = 1$ to $x = \infty$.

$$\int_1^\infty \frac{1}{x^2}\,dx$$

(a) Find an antiderivative first: $\int \frac{1}{x^2}\,dx =$

(b) $\int_1^{10} \frac{1}{x^2}\,dx =$

(c) $\int_1^{100} \frac{1}{x^2}\,dx =$

(d) $\int_1^{1000} \frac{1}{x^2}\,dx =$

(e) Do we see a pattern? What is it? $\int_1^{} \frac{1}{x^2}\,dx =$

(f) We expect $\lim_{a\to\infty} \int_1^a \frac{1}{x^2}\,dx =$

(g) Prove it!

i. First, $\int_1^a \frac{1}{x^2}\,dx =$

ii. Hence $\lim_{a\to\infty} \int_1^a \frac{1}{x^2}\,dx = \lim_{a\to\infty} \qquad\qquad =$

3. Investigate the area under $f(x) = \dfrac{1}{x^{1.1}}$ from $x = 1$ to $x = \infty$.

$$\int_1^\infty \frac{1}{x^{1.1}}\,dx$$

(a) Find an antiderivative first: $\displaystyle\int \frac{1}{x^{1.1}}\,dx =$

(b) $\displaystyle\int_1^{10} \frac{1}{x^{1.1}}\,dx =$

(c) $\displaystyle\int_1^{100} \frac{1}{x^{1.1}}\,dx =$

(d) $\displaystyle\int_1^{1000} \frac{1}{x^{1.1}}\,dx =$

(e) Do we see a pattern? What is it? $\displaystyle\int_1^{} \frac{1}{x^{1.1}}\,dx =$

(f) We expect $\displaystyle\lim_{a\to\infty}\int_1^{a} \frac{1}{x^{1.1}}\,dx =$

(g) Prove it!

i. First, $\displaystyle\int_1^{a} \frac{1}{x^{1.1}}\,dx =$

ii. Hence $\displaystyle\lim_{a\to\infty}\int_1^{a} \frac{1}{x^{1.1}}\,dx = \lim_{a\to\infty} \qquad\qquad =$

4. Investigate the area under $f(x) = \dfrac{1}{x^{1.01}}$ from $x = 1$ to $x = \infty$.

$$\int_1^\infty \frac{1}{x^{1.01}}\,dx$$

(a) Find an antiderivative first: $\displaystyle\int \frac{1}{x^{1.01}}\,dx =$

(b) $\displaystyle\int_1^{10} \frac{1}{x^{1.01}}\,dx =$

(c) $\displaystyle\int_1^{100} \frac{1}{x^{1.01}}\,dx =$

(d) $\displaystyle\int_1^{1000} \frac{1}{x^{1.01}}\,dx =$

(e) Do we see a pattern? What is it? $\displaystyle\int_1^{} \frac{1}{x^{1.01}}\,dx =$

(f) We expect $\displaystyle\lim_{a\to\infty}\int_1^{a} \frac{1}{x^{1.01}}\,dx =$

(g) Prove it!

i. First, $\displaystyle\int_1^{a} \frac{1}{x^{1.01}}\,dx =$

ii. Hence $\displaystyle\lim_{a\to\infty}\int_1^{a} \frac{1}{x^{1.01}}\,dx = \lim_{a\to\infty} \qquad =$

5. Can you draw any conclusions from these previous investigations about

(a) $\int_1^\infty \frac{1}{x^{2.1}}\, dx$, or $\int_1^\infty \frac{1}{x^3}\, dx$? Justify your answer using words and clearly labeled figures.

(b) $\int_1^\infty \frac{1}{x^{1+10^{-n}}}\, dx$? Justify.

(c) $\int_1^\infty \frac{1}{x^{1+\epsilon}}\,dx$, where ϵ is any tiny positive number? Explain.