

Wherein we investigate the behaviour of the Trapezoid Rule and Simpson's Rule with the help of an example.

1. Consider the function $f(x) = 5x^4$.
 - (a) Find the exact area under f from $x = 0$ to $x = 2$.
 - (b) Using 4 equal subintervals, estimate this area using
 - i. the Trapezoid Rule. Calculate the error in your estimate.
 - ii. Simpson's Rule. Calculate the error in your estimate.
 - (c) Compute the *theoretical* upper bound in the error committed by
 - i. the Trapezoid Rule.
 - ii. Simpson's Rule.
 - (d) Using the *theoretical* upper bounds for the errors, predict how many subintervals are needed if we have to guarantee that the error is below 10^{-6} , when using
 - i. the Trapezoid Rule
 - ii. Simpson's Rule
2. Let f be a continuous function over the interval $[1, 3]$. You are told that the following inequalities are satisfied over the entire interval $[1, 3]$.
 - $|f'(x)| < 1$
 - $|f''(x)| < 2$
 - $|f'''(x)| < 4$
 - $|f^{(4)}(x)| < 3$
 - (a) When using the Trapezoid Rule, how many subintervals are needed if we have to guarantee that the error is below 10^{-6} .
 - (b) When using Simpson's Rule, how many subintervals are needed if we have to guarantee that the error is below 10^{-6} .