

Name: _____

Please print clearly

Practise Example: Consider the ellipse $x^2 + 4y^2 = 4$. This curve can be parametrized by

$$\vec{x} = (x(t), y(t)), \quad \text{where } x(t) = 2 \cos(t), \quad y(t) = \sin(t) \quad \text{for } t \in [0, 2\pi].$$

Does this parametrization make sense? Check it out by substituing for x and y into the equation for the ellipse.

The `plot` command in Maple can be used to draw this curve:

```
plot( [2*cos(t), sin(t), t = 0..2*Pi] );
```

Note that no spaces are allowed between the dots in a `plot` command. The resulting graph is somewhat distorted, because Maple did not use the same vertical and horizontal scales. You can fix this by using

```
plot( [2*cos(t), sin(t), t = 0..2*Pi], scaling = constrained);
```

Assignment: The curve traced by a point P on a circle of radius r that rolls along the inside of another circle of larger radius R is called a *hypocycloid*. This curve can be parametrized by

$$\vec{x} = (x(t), y(t))$$

where

$$x(t) = (R - r) \cos\left(\frac{r}{R}t\right) + r \cos\left(\left(1 - \frac{r}{R}\right)t\right)$$

$$y(t) = (R - r) \sin\left(\frac{r}{R}t\right) - r \sin\left(\left(1 - \frac{r}{R}\right)t\right)$$

This formula is derived in Example 1.9.4 on p.80 of our text. See Figs 1.9.3, 1.9.4.

Take the first two **non-zero, distinct** digits of your WIN number. Set r to be the smaller of these two numbers, and R to be the larger of them. Record them here, along with your simplified equations for $x(t)$ and $y(t)$:

| R | r | $x(t)$ | $y(t)$ |
|-----|-----|--------|--------|
| | | | |

1. Use the equations of your hypocycloid, and basic knowledge of trigonometric functions to
 - (a) *predict* the range of values attained by the x and y coordinates of points on your hypocycloid. (3 pts)
 - (b) *predict* the smallest range of t (starting with $t = 0$) that should be used to get a complete picture of your hypocycloid. (3 pts)

Give clear and complete mathematical reasons for your answers. You may experiment in Maple, to provide evidence or confirmation for your answers. *But experimentation does NOT explain WHY what you observe is happening, and it will not substitute for reasoning.*

2. Plot your hypocycloid in Maple. Turn in a hard copy of your plot along with the Maple command(s) used to generate it. Make sure that the vertical and horizontal scales are the same. (3 pts)
3. Use this sheet(both sides) as the cover sheet for your assignment. Don't forget to write your name and record the values of R , r , and the simplified equations for $x(t)$, $y(t)$ in the space provided.

Should the luck of the draw leave you with an uninteresting hypocycloid (e.g. just a circle), give a reason for the uninteresting shape by analysing your parametric equations. Then choose different values for R and r and redo the assignment.