Review for Final

Disclaimer: This list is meant to be an *approximate* guide; please note that you are responsible for all the material covered in the course.

- Bring a calculator, pencil, eraser and blank paper to the final exam.

  You may also bring a 3×5 card (that's in inches). Write your name clearly on the card and submit it along with your exam. Violations will result in immediate confiscation of your card.

  The purpose of the card is to help you organize and distill the knowledge you have gained from the course.

- Needless to say, you should thoroughly go over all assigned problems (whether you were required to turn in solutions or not). The only exceptions are computer projects.

  After you have reviewed the assigned problems from a chapter, try to answer the questions from the list below that pertain to that chapter. These questions test how well you have *understood* the underlying concepts, the mathematical problems that define the subject, and the numerical methods devised to solve the problems.

1. We discussed problems that arise when computing in finite precision arithmetic, and how expressions that are algebraically equivalent in exact arithmetic can behave quite differently in finite arithmetic. Know examples of expressions that can be rewritten to have better numerical behavior, and be able to explain why and how they are better.

2. You should be able to clearly state all theorems and definitions discussed in class. Examples of theorems include: Intermediate Value Theorem, Mean Value Theorem, Rolle's Theorem, Taylor's theorem (for one variable), Mean Value Theorem for Integrals. Examples of definitions include: rates of convergence, orthogonal polynomials, cubic spline.

3. Know how to use the remainder term in the Taylor series to predict how many terms of the Taylor series are needed to compute the value of some transcendental function at a point, or over an interval, to some specified desired accuracy.

4. You should be able to give a clear statement of the main problems addressed in the course: root finding, solving systems of nonlinear equations, polynomial interpolation, numerical integration. In each case, what is given and what is being found? Be able to give a coherent description of the various solutions developed for each of these problems.

5. You should be able to clearly describe and compare two methods for root finding.

6. How can one predict the number of iterations needed by the bisection algorithm to find a root of a given function in a given interval?

7. For Newton's method for root finding, be able to give both an algebraic formula as well as a geometric explanation for the Newton iteration step. Know how to program Newton's method for a single variable on your calculator. How fast does Newton's method converge in various situations? (This is not a one line answer!)

8. Understand how the single variable Newton iteration is a special case of the multivariable iteration. What is the correspondence between the expressions in the multivariable Newton iteration and the expressions in the single variable Newton iteration? Know how to perform multivariable Newton on a system of two non-linear equations in two unknowns.

9. What conditions (on the data, the nodes, the degree of the polynomial, ...) guarantee the existence of a unique interpolating polynomial?

   What is Newton's form for the interpolating polynomial? What is Lagrange's form? Do the two forms yield different polynomials? What are the advantages and disadvantages of the Newton form vs. the Lagrange form?

   Know how to compute the interpolation polynomial (both forms) by hand, given the interpolation points.

   If p is a polynomial that interpolates f, what can be said about $|p(t) - f(t)|$? What else has to be specified about p in order for the question to be well-posed? (see Th 2, p. 315).

   What can be said in general about the convergence of interpolating polynomials as the number of nodes increases? (There are several subtle points here that you should be clear about – see Theorems 6, 7, p. 320.)

10. What is the idea behind the construction of a cubic spline? How is it different from polynomial interpolation? Are cubic splines "better" in some sense? If so, explain.

    What is a natural cubic spline?

    Is a tension spline built from polynomials?

11. What is the error in the composite Trapezoid rule? Is it $O(h)$, $O(h^2)$, $O(h^3)$, ...?

    Answer the same for Simpson's rule.

    Show how Simpson's rule is a special case of Gauss quadrature. Use this to explain the accuracy of Simpson's rule.

12. Given a desired accuracy, know how to predict how many subintervals must be used to approximate a given integral to that desired accuracy using the Trapezoid or Simpson's rule.

13. Know how to construct Newton-Cotes type quadrature formulas.

14. Give a clear statement of the Guass quadrature problem. What is the solution to this problem? Is there a connection between interpolation and quadrature, and if so, what is it?

15. Suppose $\{p_0, p_1, p_2, \ldots p_n, \ldots\}$ is a set of monic, mutually orthogonal polynomials, such that $\deg p_i = i$, for $i = 0, 1, 2, \ldots$. Explain why p_k is orthogonal to any polynomial of degree less than k.

16. You should know how to construct the first three or four Legendre polynomials from scratch. What is their connection to Gaussian quadrature? If p_4 is the Legendre polynomial of degree 4, how can it be used to construct a Gaussian quadrature formula? What would be the properties of such a formula?