Review for Midterm

Disclaimer: This list is meant to be an *approximate* guide; please note that you are responsible for all the material covered in the course.

- Bring a calculator, pencil, eraser and blank paper to the final exam.
- Needless to say, you should thoroughly go over all assigned problems (whether you were required to turn in solutions or not).

1. You should be able to clearly state and explain the meaning of the Intermediate Value Theorem; Mean Value Theorem; Taylor's theorem (for one variable): $\epsilon - \delta$ definition of a limit; rates of convergence¿
2. Understand Taylor's Theorem, and how it can be used to bound the error in polynomial approximations to non-polynomial functions.
3. Rates of convergence, Big-O, little-o notation, and how to determine if two expressions are related via Big-O, or little-o.
4. Describe the floating point, finite precision number system. Understand how numbers representable in such a system are distributed on the real number line.
5. Know how to derive relative and absolute accuracy of the four basic arithmetic operations, and understand the what the results mean.
6. We discussed problems that arise when computing in finite precision arithmetic, and how expressions that are algebraically equivalent in exact arithmetic can behave quite differently in finite arithmetic. Know examples of expressions that can be rewritten to have better numerical behavior, and be able to explain why and how they are better.
7. Give a clear description of the bisection method for root-finding. Be able to write pseudo-code for this method, explaining the issues that need to be addressed when writing code.
8. What is the rate of convergence of the bisection method, justify your answer.
9. Given a desired accuracy for the root, how can we predetermine the number of iterations needed by the bisection method to attain the desired accuracy?
10. You should know Newton's method for root finding, and be able to give both an algebraic formula as well as a geometric explanation for the Newton iteration step. Know how to program Newton's method for a single variable on your calculator. Be prepared to run Newton during the exam, and report the sequence of iterates: display digits in groups of 4 or 5, separated by spaces, mark digits that are in common between successive iterates, etc.
11. Know how simple and double roots are defined. How fast does Newton's method converge in various situations? You should be able to give a clear statement of Theorem 1 on p.85, and understand the hypothesis and the conclusion. What happens a double root, and why?
12. When does Newton's method run into problems? Describe.
13. Understand how the single variable Newton iteration is a special case of the multivariable iteration. What is the correspondence between the expressions in the multivariable Newton iteration and the expressions in the single variable Newton iteration? Know how to perform multivariable Newton on a system of two non-linear equations in two unknowns.