

Root Finding as an Application of Interpolation

Due: Tues Nov 14, at the start of class.

10 pts

Late assignments will not be accepted.

This is an adaptation of Problem 26 from Section 6.1

Include justification whenever possible.

Your methods and your logic must be clear to the reader.

Consider the function $f(x) = x - 9^{-x}$.

1. Check by drawing a suitable graph that $f(x)$ has a root in $[0, 1]$. (1 pt)
2. Find (by hand) the interpolating polynomial for f using the nodes $x_0 = 0$, $x_1 = 0.5$, $x_2 = 1$. Draw a graph f and the interpolant you found in the same graphing window. Include your observations. (3 pts)
3. Find an approximate solution to $f(x) = 0$ that lies in $[0, 1]$ by finding the roots of the interpolating polynomial you found in part(2). You may use MATLAB's `roots` function. (2 pts)
4. Use the approximate solution you found in part(3) as the starting point for Newton's method, and find the solution of the equation $x - 9^{-x} = 0$ to machine precision. How good is the approximation you found? (In future, you will not be reminded to evaluate your answers. Develop the habit of always doing this when it is possible.)

Submit your documented code and your annotated output. Aim for a balance between clarity and brevity. (4 pts)