MATH 637 **Homework I** Spring 2008

Due Date: Jan 18, 2008.

1. Trefethen & Bau: Theorem 1.3 on p. 8, lists six conditions, (b) through (g), that are equivalent to invertibility of a matrix. Prove that (d) and (e) are equivalent.

2. Trefethen & Bau, p.9: 1.1a

3. Trefethen & Bau, p.10: 1.4

Matlab Exercises

1. MATLAB has several commands that enable one to create special matrices. Use `zeros`, `ones`, `eye` and just one assignment statement to create the matrix $\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}$.

2. You can also access parts of a matrix easily in MATLAB. For example, A(8:4:20, :) will return rows 8, 12, 16, and 20 of matrix A. On the other hand, (look carefully at the syntax!) A([8, 4, 20], :) will return row 8, row 4 and row 20 of matrix A, in that order. You can access selected columns in a similar manner. (Practise commands like these on a random matrix generated using the `rand` and `randn` commands. Submatrices can be accessed by specifying selected rows and columns.)

   Suppose A is a 100×100 matrix.

   (a) Write one command to delete rows $1, 3, 5, \ldots, 99$ of A.

   (b) Write one command to delete rows 10, 12, 22 and 70 of A.

   (c) Write one command to zero out the submatrix situated in rows 3 through 10 and columns 22 through 26 of A.

3. (a) Matlab Workshop II posted on the class web page shows you how to write a function called `mytridiag` that returns the $n \times n$ tridiagonal matrix that follows the pattern suggested by $\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 6 & -1 \\ 0 & 0 & -1 & 8 \end{pmatrix}$. Write a new function that takes as input a positive integer n and returns the $n \times n$ matrix that follows the pattern suggested by $\begin{pmatrix} 3 & 1 & 0 & 0 \\ -3 & 9 & 3 & 0 \\ 0 & -3 & 27 & 6 \\ 0 & 0 & -3 & 81 \end{pmatrix}$.

4. (a) Write a script file to graph $y = \sin x$, and two of its Taylor polynomials — the linear approximation $y = x$ and the fifth degree approximation $y = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$ for $x \in [0, 2\pi]$. All three graphs must appear in the same window. Use different line

styles to distinguish the graphs. Type `more on` [1] followed by `help plot` to find out how to do this. Label the graphs using `gtext`, and label the axes using `xlabel` and `ylabel`. Give a title to your plot using `title`. A function that is useful when reading values on a graph is `grid`. Include `grid on` in your script.

(b) Investigate the error between $y = \sin x$ and its fifth degree approximation on the interval $[0, \pi/2]$. Hint: What does Taylor's theorem tell you about the truncation error?

[1] MORE ON enables paging of the output in the MATLAB command window. Type `help more` to find out more about the `more` command.