III

These difficulties which beset Reichenbach's philosophy of induction are serious, but they still leave us room to hope that it might be possible to construct a theory of induction which would justify simple enumeration without needing to make any assumptions about the uniformity of nature. Such a theory, largely free from these difficulties to which Reichenbach's view is subject, has been proposed by Professor Williams.7 His view is inviting in its simplicity, and it offers us what he takes to be a good reason for adopting and for trusting the method of induction by simple enumeration.

In his view, we must commence with the proportional, or statistical, syllogism. This is a mode of argument which, he contends, is basic to the theory of probability, being the one ultimate source from which numerical values for probabilities may be obtained (though of course once some numerical values have been introduced by means of the statistical syllogism, others may then be derived from these by means of the usual rules relating probabilities). The statistical syllogism is a mode of argument of the following form:

Of all the things that are M, \(\frac{m}{n}\) are P.

\(a\) is an M.

Therefore (with a probability of \(\frac{m}{n}\)) \(a\) is P.

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Here \( m \) and \( n \) are integers, \( a \) is an individual thing, and \('M'\) and \('P'\) are empirical predicates.

As it stands, the statistical syllogism is not an inductive mode of argument; but it can be brought to bear upon induction in the following way. Let us suppose that we have observed \( n \) ravens and found \( m \) of them to be black, the rest not black. If \( n \) is a fairly large number, then using only algebra we can prove that, whatever the total number of ravens may be (so long as they are finite in number), the great majority of the \( n \)-membered subclasses of the class of ravens differ relatively little from the whole class in regard to the fraction of their members which are black. Thus we are provided with major and minor premises for this statistical syllogism:

Of all the \( n \)-membered subclasses of the class of ravens, most differ little from the whole class in regard to the fraction of their members that are black.

This sample, whose fraction of black members is \( \frac{m}{n} \), is an \( n \)-membered subclass of the class of ravens.

Therefore (with a good probability), this sample differs little from the whole class with regard to the fraction of its members that are black.

By means of straightforward algebraic considerations, the rough notions of "most," "little," and "good" could be replaced by exact algebraic formulations or by definite numerical values if \( m \) and \( n \) are specified. And no matter how large the class of ravens may be (so long as it is finite), "most" will mean a wholesomely large percentage, "little" a pleasingly small one, and "good" a fraction nearly equal to one, all provided \( n \) is fairly large. For instance, if \( n \) is 2500, "most" can mean at least 95 per cent, while "little" will then mean not more than 2 per cent, and "high" will mean .95. Furthermore, as \( n \) increases
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without bound, "most" approaches \(\infty\) per cent as its limit, "little" approaches zero as its limit, and "good" approaches one as its limit.

It is important to note the significance of the statistical syllogism just discussed. The major premise of that syllogism is not empirical; rather its truth can be certified by algebra alone. The minor premise constitutes enumerative inductive evidence: it sums up a set of observational statements gleaned from inspection of individual ravens. The conclusion of the syllogism implies an inductive generalization: 'The fraction of all ravens that are black differs little from \(\frac{m}{n}\). Thus the statistical syllogism enables us to pass from inductive evidence to an inductive generalization as our conclusion. The final conclusion is not so simple in form as are the conclusions of the form 'All P are Q' to which some other inductive methods might purport to lead us; but for practical purposes this sort of conclusion surely is satisfactory. Here no empirical assumptions about the world have had to be made; whatever the world may be like, this argument is impeccably and necessarily valid, provided we accept the statistical syllogism itself. We seem to be offered a strong reason for embracing enumerative induction as a trustworthy and fundamental method of nondemonstrative inference.

IV

Some critics have felt that the statistical syllogism is too good to be true and that it provides somehow too easy a way out of the difficulties surrounding the logic of induction. A few of these criticisms ought briefly to be noted, because of the light they may shed upon the issue at stake.

For instance, certain critics, taking very seriously the difference between past and future, have objected that this philosophy of induction is unsound because, though the world may
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have exhibited uniformities in the past, there remains a possibility that it may not continue to do so in the future, and thus inductive reasoning may go wrong. This objection is wide of the mark, of course, for the argument mentioned in the preceding section contains and need contain no factual presupposition whatever, and certainly none about the uniformity of nature through time or otherwise. It is just this freedom from such presuppositions which is its principal merit. To be sure, the conclusion of a statistical syllogism may be false even though its premises are true; but to point this out is merely to point out that the inference is a nondemonstrative, not a demonstrative one.

In the same vein, critics have objected also that this kind of argument fails on account of its refusing to employ any empirical evidence to show that the observed sample is a “fair” one. The sample could be biased, they argue, in which case this mode of argument would yield misleading conclusions. If all the black ravens were in the top of the urn, so to speak, it might be the case that nearly all ravens are not black even though all those observed are. Such an objection as this, however, likewise neglects the fact that the statistical syllogism does not purport to guarantee the correctness of its conclusion; if it did, it would be a demonstrative, not a nondemonstrative mode of argument. What the statistical syllogism does claim to show is that its conclusion is supported by its evidence. No more should be demanded; for to demand that the conclusion be necessitated by the evidence is in effect to claim that there cannot be any nondemonstrative arguments at all. It is enough, surely, that there should be no positive reason for supposing the sample to be misleading; given the evidence that we have, we must draw whatever nondemonstrative conclusions we can from it. To contend that given evidence cannot be employed in nondemonstrative argument unless there is further positive evidence
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that the given evidence is not misleading is to embark on a vicious infinite regress, a regress which would destroy the possibility of there being any valid nondemonstrative arguments at all.

There is a further point about the statistical syllogism which deserves notice, because it may seem to militate against the usefulness of this mode of argument. This involves the matter of infinity. We noted that the statistical syllogism is applicable only where the population concerned may be assumed to be finite in size. If the population were to consist of an actually infinite number of things, then the hyperpopulation consisting of all the subclasses having the same size as the given sample would likewise be an infinite class. In that case, it would be impossible to say that most of the possible samples closely resemble the population as regards composition; for while an infinite number of them might so resemble it, an infinite number of others would fail to do so, and no definite ratio could then be said to exist between the number of those that do and the number of those that do not. Thus the statistical syllogism cannot legitimately be employed except where we are entitled to assume the population to be finite.

It may be thought that this is a serious shortcoming. Indeed, some writers on induction have blithely asserted (along with Reichenbach) that populations of empirical things in the world may be, are, or must be actually infinite in number. Some like to fancy that the class of all swans that ever exist must embrace an infinite number of birds, that there are a limitless number of inhabitants of Africa, and so on. All who regard the spatio-temporal world as this full of things will have to regard the statistical syllogism as yielding an inadequate rule of induction, since it cannot serve to make probable any generalizations about such prodigious classes.

But is this prodigality justified? Need we seriously suppose
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that classes of things in the world may be so big? Surely this is a needless metaphysical assumption. For to say of any class of empirical things that it contains an actually infinite number of members is to make an assertion utterly devoid of empirical significance. This follows from the fact that no possible tests or observations could conceivably establish the statement; we might have empirical evidence that there exist at least a million swans, or at least a billion; but in the nature of the case, no evidence could establish that the number of swans is greater than every finite number. The supposition that the number of actual members of some class of empirical individuals is infinite is an untestable metaphysical supposition, and need not be taken seriously, so far as empirical knowledge is concerned. There are less things in heaven and earth than are dreamed of in some philosophies.