Eight Versions of Bayes's Theorem

Name	Formula	Comments
Simple form	$P(H\|E) = \frac{P(H)\,P(E\|H)}{P(E)}$	This is the simplest form, derivable from the conjunction rule. When H ⊨ E, it reduces to P(H|E) = P(H) / P(E), an important special case
Explicit form	$P(H\|E) = \frac{P(H)\,P(E\|H)}{P(H)\,P(E\|H) + P(\sim H)\,P(E\|\sim H)}$	This is obtained from the simple form by expanding P(E) according to the theorem on total probability (TTP)
General form	$P(H_1\|E) = \frac{P(H_1)\,P(E\|H_1)}{P(H_1)\,P(E\|H_1) + \ldots + P(H_n)\,PE\|H_n)}$	This is again obtained from the simple form by the TTP on the assumption that $H_1, \ldots, H_n$ form a partition
Sigma form	$P(H_1\|E) = \frac{P(H_1)\,P(E\|H_1)}{\Sigma_i\,P(H_i)\,P(E\|H_i)}$	This is a compact way of writing the general form, of which it is a notational variant
Canceled form	$P(H\|E) = \frac{1}{1 + \frac{P(\sim H)}{P(H)} \times \frac{P(E\|\sim H)}{P(E\|H)}}$	This is obtained from the explicit form by dividing the numerator and the denominator by P(H)P(E|H) and collecting terms. It is very useful when we replace E with the testimony of multiple independent witnesses
Odds form	$\frac{P(H\|E)}{P(\sim H\|E)} = \frac{P(H)}{P(\sim H)} \times \frac{P(E\|H)}{P(E\|\sim H)}$	This is useful because, as in the canceled form, the term P(E) has dropped out. The likelihood ratio on the right is the "Bayes factor," the term by which the prior odds are multiplied to obtain the posterior odds
Relative odds form	$\frac{P(H_1\|E)}{P(H_2\|E)} = \frac{P(H_1)}{P(H_2)} \times \frac{P(E\|H_1)}{P(E\|H_2)}$	This generalization of the odds form is useful when we do not have a partition but we wish to know the relative impact of E on two hypotheses.
Compound odds form	$\frac{P(H\|A\&B)}{P(\sim H\|A\&B)} = \frac{P(H)}{P(\sim H)} \times \frac{P(A\|H)}{P(A\|\sim H)} \times \frac{P(B\|H)}{P(B\|\sim H)}$	This form allows combination of evidence, assuming that ±H screens off A from B. Without this assumption, the probabilities in the final term would have to be conditional on A as well. Given this assumption, we can combine evidence by multiplying the prior odds by successive Bayes factors.