Chapter Six

The Problem of Deduction

How can one be justified in taking the principles of deductive reasoning themselves to be truth-preserving? Surely any cogent argument for the validity of deductive reasoning must use deduction. But will not any such argument, however rigorous, be epistemically circular? And if one is not justified by argument, then how? It appears that in criticizing epistemic circularity, we may have opened ourselves to a damaging *tu quoque*.

William Alston articulates the problem in terms of an attempt to prove the reliability of deduction:

> As for deduction, it quickly becomes obvious that anything that would count as showing that deduction is reliable would have to involve deductive inference and so would assume the reliability of deduction. Just try it. For example, for the case of the propositional calculus, we can demonstrate the reliability of any inference form by truth tables. But doing so is itself a case of deduction.¹

Alston finds this conclusion disquieting, since it can be generalized to all “sources of belief.” Our epistemic principles are finite in number; like the inhabitants of a small hamlet, they cannot take in each other’s epistemic laundry more than a few times without creating a circle. Faced with this problem we cannot simply suspend belief, for this would mean suspending all beliefs, and that is not only practically unfeasible but also self-defeating. We are, Alston says, “ineluctably engaged in forming beliefs in ways we cannot non-circularly show to be reliable. And that sticks in our craw.”²

What sticks in Alston’s craw is not, to others, particularly disturbing. According to Plantinga, our inability to justify deduction is of a piece with the rest of our cognitive life and should be no cause for alarm. Of course deduction cannot be “credentialed” by a derivation of its reliability from any independently certifiable source of beliefs; even God could not do better than an epistemically circular justification of deduction.³ What we must fall back on is the *spontaneity* of our inclination to believe, despite our awareness that this is, from the standpoint of traditional skepticism, a bruised reed:

> When we contemplate the corresponding conditional of *modus ponens*, we just find ourselves with this powerful inclination to believe that this proposition is true, and indeed couldn’t be false. But (as we also know) such inclinations are by no means infallible. We really don’t have any reasons or


²Ibid., p. 121.

grounds for this belief; we simply, so to say, start with it.4

Some philosophers find epistemic circularity at the level of allegedly a priori knowledge positively welcome. Susan Haack concludes a detailed critique of deductive justifications of deduction with a distinct air of satisfaction:

The moral of this paper might be put, pessimistically, as that deduction is no less in need of justification than induction; or, optimistically, as that induction is in no more need of justification than deduction. ... Those of us who are sceptical about the analytic/synthetic distinction will, no doubt, find these consequences less unpalatable than will those who accept it. And those of us who take a tolerant attitude to nonstandard logics -- who regard logic as a theory, revisable, like other theories, in the light of experience -- may even find these consequences welcome.5

The easiest way to see the difficulty posed when one attempts to justify the rules of deductive inference is to look at the structure of a plausible justificatory argument. Taking a cue from Alston, let us try to construct an argument from the truth table for conditional propositions to the inevitable truth-preserving nature of modus ponens. First we display the relevant information in a truth table:

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>(P \rightarrow Q)</th>
<th>((P &amp; (P \rightarrow Q)) \rightarrow Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
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Now we are in a position to argue as follows:

(A) 1. Every line of the truth table that assigns truth to both P and (P \rightarrow Q) also assigns truth to Q.

2. But if this is so, then whenever P and (P \rightarrow Q) are true, Q must also be true.


Hence,

3. Whenever P and (P → Q) are true, Q must also be true.

Here, line 3 ascribes to modus ponens the property that is the very definition of deductive validity, so we seem to have achieved our goal. Alas! (A) itself is an instance of modus ponens, and the argument therefore runs afoul of our ban on epistemic circularity. The extension of this problem to roundabout deductive justifications of deduction is straightforward. For every rule of inference used in the justification, a thoroughgoing deductive skeptic will demand an independent justification. If we are resourceful enough never to repeat an argumentative pattern, we can prolong the discussion but never resolve the issue to the skeptic’s satisfaction; if we are not so resourceful, we will eventually repeat a form, creating an epistemic circle.

To the student of logic, this raises some disquieting questions about the fundamental results of logical metatheory. One of the showpieces of a rigorous course in elementary logic is the demonstration that both the propositional calculus and standard first-order predicate calculus are sound and complete. Put briefly and somewhat colloquially, a proof of soundness assures us that our rules of inference are not too strong, and a proof of completeness assures us that they are not too weak. In view of the inevitable use of inferences in these metatheoretical pursuits, as Michael Friedman notes, this procedure is “in an important sense circular.” Nevertheless, he goes on, it does nonetheless provide an important kind of justification for deductive methods of inference. It shows that there is a desirable harmony between the methods we use in practice and our conception of what the point of those methods is, namely, preservation of truth from premises to conclusion. ... The completeness theorem, therefore, accomplishes something significant despite its circularity.6

But suppose that unbeknownst to us our rules of inference are too strong, allowing us to derive conclusions that we would have been unwilling to sanction if we had been sufficiently vigilant. Then following those very patterns of inference we might in all innocence derive things that did not, in point of logical fact, really follow from our premises; and why should we not find, among the results that we might thus derive, the soundness and completeness of our system of logic?

Consider the following argument for the truth-preserving nature of the well-known deductive fallacy of affirming the consequent, known by weary logic professors as modus morons:

(M) 1. If the argument from (P → Q) and Q to the conclusion P is valid, then (P v ~P) is a tautology.

2. \((P \lor \neg P)\) is a tautology.

Hence,

3. The argument from \((P \rightarrow Q)\) and \(Q\) to the conclusion \(P\) is valid.

The premises of \((M)\) are true; and though its form is an instance of \textit{modus morons} itself, Friedman’s line of reasoning would encourage us to let that pass. Yet it ought to go without saying that this argument does not accomplish anything significant and certainly does not constitute a justification for affirming the consequent. If this argument gives us a feeling of “desirable harmony” between the practice of affirming the consequent and the truth-preserving nature of deductive reasoning, so much the worse for us.

The moral to be drawn here is not that logical metatheory has no point or that soundness and completeness are not, after all, desirable properties. But Friedman is simply mistaken to think that such arguments can provide, as he claims they can, “an important kind of justification for deductive methods of inference.” Rather, the moral is a conditional one: if we cannot trust the reasoning used specific forms of reasoning used in our metatheoretic arguments, then our derivation of soundness or completeness is pointless. Far from giving us a justification for deductive inference, such a derivation is an exercise in blind faith.

\textit{Intuition, demonstration, and the status of metatheory}

In the previous chapter we expounded and adapted a Lockean theory of intuition to analytic knowledge generally. Logical metatheory requires yet a further Lockean concept, the notion of demonstration. When we turn to the relation of intuition and demonstration, we find Locke adamantly insisting on the primacy of the former. Both in the \textit{Essay} and in the controversy with Stillingfleet he maintains that the syllogism, though not to be despised, is itself dependent on intuition – that the latter is, in Aaron’s felicitous phrase, the “cognitive core of reasoning as inferring.” A famous passage added to the fourth edition of the \textit{Essay} spells this out in detail.

But God has not been so sparing to men as to make them barely two-legged creatures, and left it to Aristotle to make them rational, i.e., those few of them that he could get so to examine the grounds of syllogisms, as to see that, in above three score ways that three propositions may be laid together, there are but about fourteen wherein one may be sure that the conclusion is right; and upon what grounds it is, that, in these few, the conclusion is certain, and in the other not. God has been more bountiful to mankind than so. He has given them a mind that can reason, without being instructed in methods of syllogizing: the understanding is not taught to reason by these rules; it has a native faculty to perceive the coherence or incoherence of its ideas, and can range them right, without any such perplexing repetitions.


\footnote{Locke, \textit{Essay}, IV, xvii, 4, p. 391.}
Despite some strong language, Locke does not disparage the syllogism tout court. In his polemical writings against John Edwards just two years before the insertion of this passage in the fourth edition of the *Essay*, Locke calls syllogism “the true touchstone of right argument” and intimates that Edwards’s work fares poorly when measured against such a standard. The point rather is that the study of syllogism cannot supply a grasp of conceptual relations if such grasp is lacking in the first place, and that often the purposes to which syllogistic reasoning is put, whether persuasive, pedagogic, or dialectical, are better served by indicating the containments and connections of concepts directly.

In chapter 5 we discussed various intuitions of implication as reasonable extensions of the Lockean notion of conceptual containment; these intuitions form the basis for the construction of demonstrations. In demonstration, formal proofs are built by the application of rules that licence the transition from a set of formulas to a formula. The application of a Lockean epistemology to formal logic will involve the certification of some such rules in virtue of the meanings of the component terms.

The most natural way to display those meanings is to use truth tables. But as we saw above, to argue from the truth table of the corresponding conditional for *modus ponens* to the truth-preserving nature of this inference by means of the schema (A) would entangle us in epistemic circularity. Yet every teacher of logic knows that some truth-table arguments are both possible and pedagogically helpful. How can we reconcile epistemic rigor with pedagogic practice?

The key term here is *display*. Truth tables are an aid for the logically myopic, not a cure for the logically blind. We display the meanings of connectives in a matrix in order to clarify their use – to stress, for example, that the formal symbol ‘&’ is not being used in a temporally sensitive way as its English counterpart ‘and’ sometimes is, that the symbol for ‘or’ is being used inclusively, that negation cannot here be doubled as an intensifier. There is nothing epistemically circular about this procedure. But any arguments we make from truth tables to the virtues of rules of inference presuppose that the reader already grasps the entailments in question; and when those entailments turn on the logical properties of terms like ‘and,’ ‘or,’ and ‘not,’ we are presupposing that the reader has a sufficient grasp of those concepts to see the entailment.

As myopia comes in degrees, so, as we have repeatedly emphasized, does logical perspicacity. There may be the occasional genius who can see at a glance that a complex formal system is sound or complete, but for most of us this is a bit of knowledge arrived at by demonstration and not something apprehended in a direct intuition. And this is precisely why we need soundness and completeness proofs. Step by intuitively certified step we can lay out the argument that our rules of inference are not too strong or too weak. Demonstrations, then, are in effect *explications* which draw out at least some conceptual entailments of premises for those who can grasp clearly and distinctly those premises by themselves.

Locke held that demonstration led to certainty just as did intuition, but this claim does

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require a caveat. Whether or not demonstration brings rational certainty will depend in any given case upon whether memory is involved. In some cases the very act of demonstration may show conceptual entailments so clearly that, once the subject sees the demonstration, he is then able to hold the entire chain of reasoning in his mind at once without reliance upon memory. Such a mental act will be closely akin to intuition, although it will be, so to speak, complex rather than simple, in that it will consist of the immediate perception of a set of logical relations that reveal conceptual connections rather than of a single conceptual relationship. On the other hand, if the demonstration can only be seen partially at any one time, the subject will be relying on memory and so the resulting conclusion will not be entirely a matter of a priori knowledge. However, as we discussed in both chapter 2 and chapter 5, this fact does not impugn the infallible nature of a priori knowledge as such.

What difference does the Lockean analysis make to the epistemic question raised about the status and putative epistemic circularity of logical metatheory? The rationality of the modes of reasoning used in an epistemically cogent consistency or completeness proof must themselves be intuitable – fixed points on which the proof can turn. The status of those fixed points is not in question here, any more than it is in question when we use them to explain how the truth-tables convey the meaning of the logical connectives. Hence, there is no metaregress and no epistemic circularity, for intuitable truths satisfy the requirements of the modal principle discussed in chapter 4.

For example, in Nagel and Newman’s charming proof of absolute consistency for an axiomatized system S of sentential logic,11 we find the following argument:

(P) 1. If there is a formula expressible but not derivable within system S, then system S is consistent.

2. There is a formula expressible but not derivable within system S.

3. System S is consistent.

This would be epistemically circular if it were offered as a reason for the validity of modus ponens; for (P) is itself an instance of modus ponens, which is a rule of the system under consideration. But this is not the point of (P). Rather, the metalogician offers (P) on the understanding that the reader can see, intuitively, the legitimacy of modus ponens.

That is not to say that the metatheoretic arguments lack cogency. Rather, they are cogent for those who are able to understand them as they are intended. Nagel and Newman’s proof establishes the absolute consistency of S. But the consistency of S is not something any ordinary person would plausibly find obvious, and few even among logicians could hold it in the mind in a single act of intuition. This is why, in the fuller version of the proof, the various steps

11Nagel and Newman, Godel’s Proof (New York: New York University Press, 1958), pp. 50-1. Note that the system under consideration is strong enough to permit the derivation of any formula from two contradictory formulas.
undergirding the premises are chained together in a demonstration: the verification that each axiom is tautologous, the proof by mathematical induction that each rule of inference transforms tautologies into tautologies, the demonstration that from two contradictory formulas every formula is derivable, the exhibition of a statement that is not a tautology. The proof proceeds by just those steps that we require with our limited logical perspicacity. And as we understand them and the way that they fit together, we have demonstrative knowledge in virtue of our ability to grasp the cogency of the fundamental steps.

Are there people so inept logically that they cannot intuit the validity of modus ponens? In an academic world where postmodernists are viewed as intellectual superstars, it would be rash to be sure that there are not. But it would be equally foolish to suggest that they could profit from a consistency proof, or that the genuine cogency of modus ponens is in any way called into question by the existence of such cognitive cripples. Perhaps persistent work on logic could act as therapy to improve their cognition. But unless and until they develop the ability to recognize the validity of basic rules of inference, logical metatheory must remain for them a closed subject.12

The notion of logical intuition is what is missing from Friedman’s account of metatheory, and the lack of it is what makes his account dissatisfying. Only with a concept of a priori intuition do we have the resources to say why an alleged proof of anything using modus morons is worthless, since the validity of modus morons is not intuitable and, indeed, the fact that it is not valid is intuitable, if only by clearly grasping the relevance of a counterexample. And only with this account do we have the resources to explain how the a priori answers the problem of the metaregress for deductive logic. The fact that modus ponens is a valid form of inference is itself an intuitable analytic truth, as are similar principles for other deductive argument forms. Such intuitions will involve understanding concepts like validity, truth, and material implication.

The fact that we can demonstrate the consistency or completeness of deductive systems by metatheoretic proofs does not mean that such proofs are needed to defend basic logical steps like modus ponens, as if the rationality of these steps were intrinsically dubitable and stood in need of an argument to show their validity. Rather, metatheory demonstrates the properties of entire systems by way of steps whose validity we are able to grasp directly. Intuitable logical truths thus provide the metafoundational stopping places for deduction.

12On the other end of the spectrum, we may in rare cases encounter a principle of inference so recondite that no human being is sure whether it is valid. In 1935, just four years after the publication of Kurt Godel’s famous incompleteness theorem, Gerhard Gentzen proved the consistency of the full system of elementary number theory. Gentzen’s proof, however, appeals to a principle of inference known as transfinite induction that belongs neither to first order logic nor to elementary number theory itself. The embarrassing result is that no one is sure what to make of the proof. It is valid (and hence number theory is consistent) provided that transfinite induction is legitimate. But to date no one has found a convincing argument in intuitively certifiable steps that shows the validity of transfinite induction.