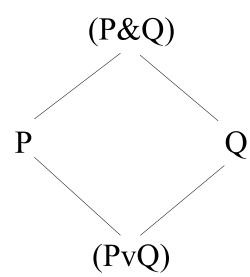
Grad Logic Extra Credit Assignment

1. Suppose that in a certain system of “logic” (not ours), it is not true that every tautologous sequent is derivable. What does this fact absolutely determine about the consistency and completeness of the system? What, if anything, does this fact leave open as a possibility but not determine? (Assume that in this system, from an inconsistency, everything follows.)

2. Construct a lattice! This is easier than it sounds. In the following list of propositions, some of the propositions entail others (that is, a sequent of the form ‘$\varphi \vdash \psi$’ holds), but in other cases there is no entailment from one to the other. Place the logically strongest statement at the top, and draw a line downward to the next (weaker) statements entailed by it, then down from those to the statements entailed by those, until finally all of the paths come back together to touch the very weakest statement at the bottom of the lattice – the one entailed by everything.

Here, for example, is a lattice for the set P, Q, (P&Q), (PvQ):



The list below does contain a strongest and a weakest statement. (For the purposes of this exercise, I am suspending the requirement that formulas have all internal parentheses.)

P, Q, R, (P&Q), (P&R), (Q&R), (P&Q&R), (PvQ), (PvR), (QvR), (PvQvR)

3. Derive the following sequents. You may use the ten basic rules and, in addition, any of the following short-cut sequents, provided that the sequent you are proving is not a substitution instance of them, in which case that short-cut is off limits for that proof:

<u>Shortcuts</u>

1) $-(P \,\&\, Q) \vdash (-P \vee -Q)$
2) $-(P \vee Q) \vdash (-P \,\&\, -Q)$
3) $-(P \rightarrow Q) \vdash (P \,\&\, -Q)$
4) $(P \rightarrow Q) \vdash (-P \vee Q)$
5) $(P \vee Q), -P \vdash Q$ or $(P \vee Q), -Q \vdash P$

<u>Sequents to prove</u>

(a) $(-P \rightarrow Q) \vdash (P \vee Q)$
(b) $(P \rightarrow (Q \rightarrow R)) \vdash ((P \,\&\, Q) \rightarrow R)$
(c) $(P \rightarrow (Q \rightarrow R)) \vdash (Q \rightarrow (P \rightarrow R))$
(d) $-(P \vee -P) \vdash Q$
(e) $\vdash (((P \rightarrow P) \rightarrow P) \rightarrow P)$