

Summary of the Fundamental Rules and Definitions of Probability

Rule	Unconditional form	Conditional form (assuming $P(C) \neq 0$)
Normality	$P(A) + P(\sim A) = 1$	$P(A C) + P(\sim A C) = 1$
Multiplication (Conjunction)	$P(A \& B) = P(A B) P(B)$ In the special case where A and B are independent, $P(A \& B) = P(A) P(B)$.	$P(A \& B C) = P(A B \& C) P(B C)$ In the special case where A and B are conditionally independent given C, $P(A \& B C) = P(A C) P(B C)$
Addition (Disjunction)	$P(A \vee B) = P(A) + P(B) - P(A \& B)$	$P(A \vee B C) = P(A C) + P(B C) - P(A \& B C)$
Total Probability (Elimination)	$P(A) = P(B) P(A B) + P(\sim B) P(A \sim B)$ $= P(A \& B) + P(A \& \sim B)$ This can also be written $\sum_i P(A, B_i) = P(A)$, where we are summing over the possible values i can take. If B_1, \dots, B_n form a partition, then $P(A) = \sum_i P(B_i) P(A B_i)$, that is, the sum over the possible values for each of the elements of the partition. Written out the long way, this would be $P(A) = P(B_1) P(A B_1)$ $+ P(B_2) P(A B_2)$ $+ \dots$ $+ P(B_n) P(A B_n).$	$P(A C) = P(B C) P(A B \& C) + P(\sim B C) P(A \sim B \& C)$ $= P(A \& B C) + P(A \& \sim B C)$ If B_1, \dots, B_n form a partition relative to C, then $P(A C) = \sum_i P(B_i C) P(A B_i \& C)$.

Definition	Unconditional form	Conditional form
Conditional Probability	$P(A B) = \frac{P(A \& B)}{P(B)}$	$P(A B \& C) = \frac{P(A \& B C)}{P(B C)}$
Independence	A and B are <i>independent</i> iff $P(A B) = P(A)$ Since independence is a symmetric relation, we could also write $P(B A) = P(B)$. In standard formulations of the probability calculus, $P(A B)$ is defined only when $P(B) > 0$.	A and B are <i>conditionally independent given C</i> iff $P(A B \& C) = P(A C)$ In this case we sometimes say that C <i>screens off</i> B with respect to A; some authors use this terminology only if A and B are conditionally independent given C but not independent <i>simpliciter</i> .
Certainty and Entailment	If A is a certainty, then $P(A) = 1$ All logical truths are certainties, but it is controversial whether all certainties must be logical truths.	If B entails A, then $P(A B) = 1$ If A is entailed by a logical truth B, then A is itself a logical truth and therefore a certainty.
Exclusion	A and B are mutually exclusive iff A entails $\sim B$. It will follow from this that $P(A \& B) = 0$.	A and B are mutually exclusive relative to C if A & C entails $\sim B$. It will follow from this that $P(A \& B C) = 0$.
Partition	A set of propositions A_1, \dots, A_n forms a partition iff both (a) when $i \neq j$, A_i and A_j are mutually exclusive, and (b) A_1, \dots, A_n jointly exhaust the probability space. This second requirement entails that $P(A_1 \vee \dots \vee A_n) = 1$ but is not entailed by it.	A set of propositions A_1, \dots, A_n forms a partition with respect to C iff both (a) when $i \neq j$, A_i and A_j are mutually exclusive relative to C, and (b) A_1, \dots, A_n jointly exhaust the probability space conditional on C, or, equivalently, C entails the disjunction $A_1 \vee \dots \vee A_n$.