Summary of the Fundamental Rules and Definitions of Probability

Rule	Unconditional form	Conditional form (assuming P(C) ≠ 0)
Normality	P(A) + P(~A) = 1	P(A|C) + P(~A|C) = 1
Multiplication (Conjunction)	P(A&B) = P(A|B) P(B) In the special case where A and B are independent, P(A&B) = P(A) P(B).	P(A&B|C) = P(A|B&C) P(B|C) In the special case where A and B are conditionally independent given C, P(A&B|C) = P(A|C) P(B|C)
Addition (Disjunction)	P(AvB) = P(A) + P(B) – P(A&B)	P(AvB|C) = P(A|C) + P(B|C) – P(A&B|C)
Total Probability (Elimination)	P(A) = P(B) P(A|B) + P(~B) P(A|~B) = P(A&B) + P(A&~B) This can also be written Σ_iP(A,B_i) = P(A), where we are summing over the possible values i can take. If B_1, ... , B_n form a partition, then P(A) = Σ_i P(B_i)P(A|B_i), that is, the sum over the possible values for each of the elements of the partition. Written out the long way, this would be P(A) = P(B_1) P(A|B_1) + P(B_2) P(A|B_2) + ... + P(B_n) P(A|B_n).	P(A|C) = P(B|C) P(A|B&C) + P(~B|C) P(A|~B&C) = P(A&B|C) + P(A&~B|C) If B_1, ... , B_n form a partition relative to C, then P(A|C) = Σ_i P(B_i|C) P(A|B_i &C).

Definition	Unconditional form	Conditional form
Conditional Probability	$P(A\|B) = \dfrac{P(A\&B)}{P(B)}$	$P(A\|B\&C) = \dfrac{P(A\&B\|C)}{P(B\|C)}$
Independence	A and B are *independent* iff P(A|B) = P(A) Since independence is a symmetric relation, we could also write P(B|A) = P(B). In standard formulations of the probability calculus, P(B|A) is defined only when P(B) > 0.	A and B are *conditionally independent given C* iff P(A|B&C) = P(A|C) In this case we sometimes say that C *screens off* B with respect to A; some authors use this terminology only if A and B are conditionally independent given C but not independent *simpliciter*.
Certainty and Entailment	If A is a certainty, then P(A) = 1 All logical truths are certainties, but it is controversial whether all certainties must be logical truths.	If B entails A, then P(A|B) = 1 If A is entailed by a logical truth B, then A is itself a logical truth and therefore a certainty.
Exclusion	A and B are mutually exclusive iff A entails ~B. It will follow from this that P(A&B) = 0.	A and B are mutually exclusive relative to C if A&C entails ~B. It will follow from this that P(A&B|C) = 0.
Partition	A set of propositions A_1, ... , A_n forms a partition iff both (a) when i≠j, A_i and A_j are mutually exclusive, and (b) A_1, ... , A_n jointly exhaust the probability space. This second requirement entails that P(A_1 v ... v A_n) = 1 but is not entailed by it.	A set of propositions A_1, ... , A_n forms a partition with respect to C iff both (a) when i≠j, A_i and A_j are mutually exclusive relative to C, and (b) A_1, ... , A_n jointly exhaust the probability space conditional on C, or, equivalently, C entails the disjunction A_1v ... v A_n.

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