Power Factor Correction
ECE 2100 Circuit Analysis
version 8 January 2022

Equipment and Supplies

<table>
<thead>
<tr>
<th>Equipment</th>
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<tbody>
<tr>
<td>oscilloscope</td>
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<tr>
<td>waveform generator</td>
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<tr>
<td>2.4kΩ 1/4W resistor</td>
</tr>
<tr>
<td>100Ω 1/4W resistor</td>
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<tr>
<td>39mH inductor</td>
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<tr>
<td>capacitor substitution box</td>
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</tbody>
</table>

Learning Outcomes

Students will:
1. Apply the phasor concept to the analysis of an AC circuit;
2. Compute the value of a capacitor to achieve a unity power factor for an inductive load; and
3. Verify power factor correction in the time and phasor domains in simulation and the lab.

Pre-Laboratory Assignment (STEPS 1-14)

Phasor Domain Analysis and Simulation of Series RL Circuit

Hand Analysis

1. **Use RMS values for all phasors.**
   Note that ammeters and voltmeters measure RMS values of AC waveforms.

2. Ignoring capacitor $C_1$, compute the following phasors for the circuit of Figure 1 (note the frequency is 10kHz and thus $\omega=2\pi10kHz$): $V_S$, $I_S$, $V_R$, and $V_L$.
   Place values in Table 1.
   Neatly plot phasors $V_S$, $I_S$, $V_R$, and $V_L$.

3. Verify that the following angles are correct on your phasor plot:
   a. $\angle V_R - \angle I_S = 0$; and
   b. $\angle V_L - \angle I_S = 90^\circ$.
   Also check that $V_S = V_R + V_L$.
   Enter the indicated angles in Table 1.

4. Compute the equivalent impedance $Z_{in}$ of Figure 1 and record value in Table 1.
   $Z_{in}$ is the impedance of $R_1$ in series with $L_1$ at 10kHz.
   In a motor circuit model $R_1$ corresponds to winding resistance and $L_1$ winding inductance (other motor effects are neglected here).

5. Use the angle of $Z_{in}$ to compute the power factor of the load ($R_1$ in series with $L_1$).
6. Find the complex power $S_{src}$ of the source voltage $V_S$ and the complex power of the load $S_{load}$. Record values in Table 1. Do they add to zero? They must!

7. Use the angle of $S_{load}$ to verify the load power factor (must match the result of pre-lab step 5).

![Power Factor Correction Circuit](image)

**Figure 1. Power Factor Correction Circuit**

<table>
<thead>
<tr>
<th>USE RMS VALUES</th>
<th>Hand Analysis</th>
<th>Simulation</th>
<th>Experimental</th>
</tr>
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<tbody>
<tr>
<td>$V_S$</td>
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<td></td>
<td></td>
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<tr>
<td>$I_S$</td>
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<td></td>
<td></td>
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<tr>
<td>$V_R$</td>
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<tr>
<td>$V_L$</td>
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<tr>
<td>$\angle V_R-\angle I_S$</td>
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<td>$\angle V_L-\angle I_S$</td>
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<tr>
<td>$\angle V_S-\angle I_S$</td>
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<td></td>
</tr>
<tr>
<td>$Z_{in}$</td>
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</tbody>
</table>

$S_{load} = V_S (I_S)^*$

(load complex power)

$S_{src} = -V_S (I_S)^*$

(source complex power)

Table 1. Results Table (Without Power Factor Correction)
Simulation

8. Simulate the circuit of Figure 1. Note that results are not saved until after 100us. This ensures circuit voltages and currents are in the sinusoidal steady state.

9. Use your simulation to find each voltage and current phasor and angles of Table 1. Put results in table in ‘Simulation’ column.

An example computation of the phase angle the angle $\angle V_L - \angle I_S$ is shown in Figure 2. CURSOR 1 is used to measure the time of the peak value of V(2) (this is $V_L$). CURSOR 2 is used to measure the time of the nearest peak of I(L1) (this is $I_S$).

Since the peak of $V_L$ occurs earlier in time than the peak of $I_S$, $V_L$ LEADS $I_S$. Thus $\angle V_L - \angle I_S$ is positive.

The time difference between the peaks is $\Delta t = 37.569\text{us} - 12.569\text{us} = 25\text{us}$.
You could zoom into the plot to improve measurement resolution, but it turns out that this is the theoretical value.

The time of one complete cycle (also determined using cursors) is $T = 100\text{us}$.

Thus the angle is $\Delta t / T \times 360^\circ = 90^\circ$.

Figure 2. Using cursors to measure peak values used to compute a phase angle.
10. Compute $Z_{in} = \frac{V_S}{I_S}$ and enter in Table 1.

Use the angle of $Z_{in}$ to compute the power factor of the load (R1 in series with L1).

Use your simulation results to compute $S_{load}$ and $S_{src}$. Put results in Table 1.

All hand analysis and simulation results must be in close agreement.

**Power Factor Correction**

11. Compute the value of $C1$ needed for $C1$ to have complex power $S_C = -\text{Im}[S_{LOAD}]$.

For example, if $S_{LOAD} = 1 + j \text{ VA}$ then $S_C = -j \text{ VA}$. Enter value into Table 2.

*Now the new load, consisting of R1, L1, and C1, has no reactive power and therefore the load power factor is now one.*

Also note that the voltages across and currents through R1 and L1 will not change and we do not need to cut any wires to insert C1 – the capacitor goes in parallel with R1 and L1. This is an important practical consideration.

12. With $C1$ added to the circuit, compute Table 2 ‘Hand Analysis’ column quantities.

Note that $V_S$ and $I_S$ are now in phase.

Neatly plot phasors $V_S$, $I_S$, $V_R$, $V_L$ and $V_C$.

13. Verify phasors $V_S$ and $I_S$ using LTspice®. Complete the ‘Simulation’ column of Table 2. ($Z_{in}$, load power factor, $S_{load}$, and $S_{src}$ are computed using simulated values of $V_S$ and $I_S$).

14. Bring an electronic copy of your simulation files to lab.

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<tr>
<td>$\angle V_S - \angle I_S$</td>
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<tr>
<td>$Z_{in}$</td>
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<tr>
<td>load power factor</td>
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<tr>
<td>$S_{load} = V_S(I_S)^*$</td>
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<td>(load complex power)</td>
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<tr>
<td>$C1$</td>
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</table>

Table 2. Results Table (With Power Factor Correction)
**Procedures**

1. **Use RMS values for all phasors.**

2. With all equipment off and the waveform generator amplitude control turned to zero, construct the circuit of Figure 1 WITHOUT the capacitor. Insert a 100Ω current sampling resistor in series between L1 and ground.

   Connect channel 1 of the oscilloscope to $v_s(t)$. Connect channel 2 of the oscilloscope across the current sampling resistor to measure $v_{samp}(t)$ (being careful to observe the passive sign convention). You will use $v_{samp}(t)$ to display the current $i_s(t)$ by dividing $v_{samp}(t)$ by the value of the current sampling resistor (100Ω).

3. Turn on the waveform generator and oscilloscope. Configure the waveform generator to produce a 10V peak sinusoidal voltage with zero DC offset at a frequency of 10 kHz.

4. Measure $V_S$ and $I_S$. Put results in Table 1 ‘Experimental’ column. Compute the experimental $Z_{in}$, load power factor, $S_{load}$, and $S_{src}$ and add to Table 1.

5. Turn off the waveform generator. Insert $C1$ computed in pre-lab step 11 into the circuit.

6. Turn on the waveform generator and check that $v_s(t)$ is still a 10V peak 10 kHz sinusoid. Measure the peak amplitude of $v_s(t)$ and $v_{samp}(t)$. Compute the peak value of $i_s(t)$ by dividing $v_{samp}(t)$ by 100.

   The phase shift between $v_s(t)$ and $i_s(t)$ **SHOULD BE ZERO** indicating that only real power is being drawn from the source.

   Determine $V_S$ and $I_S$. Put results in table ‘Experimental’ column of Table 2. Compute the experimental $Z_{in}$, load power factor, $S_{load}$, and $S_{src}$ and add to Table 2.

7. Vary $C1$ 10% above the computed value and record how the phase shift changes. Repeat for 10% below the computed value of $C1$.

**Analysis**

1. Provide a phasor diagram for the capacitor, inductor, and resistor complex powers for two cases:
   
   a. $C1=0$, and
   
   b. $C1$ corresponding to the unity power factor case.

2. Why does the resistor power remain the same before and after $C1$ is added even though the current provided by the voltage source changes? Why might this be desirable?

3. Using a hand analysis find the percentage error in the magnitude and phase of $i_s(t)$ introduced by the current sampling resistor for the $C1=0$ case.

4. Perform an internet search to find the price of a power factor correction capacitor. Provide the
model number, its rated value in kVA (usually listed as KVAR where R stands for reactive), and website address.

Credits and Copyright

Adapted from material developed by current and former ECE faculty, including J. Kelemen and F. Severance. Modified based on input from ECE laboratory instructors, particularly M. El Yabroudi, S. Hajian, S. Masihi, and M. Panahi. S. Durbin contributed to the development of this laboratory.

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