2: Passive Bandpass Filter Design

ECE 3200 Electronics II
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Reference


Objectives

1. To generate a design based on a set of specifications.
2. To demonstrate the measurement and application of Bode plots in circuit design and analysis.
3. To improve and further develop an ability to effectively communicate technical information via a written report.

Pre-Laboratory Assignment

(ONE PRE-LAB ASSIGNMENT PER GROUP — WORK TOGETHER)

Consider the circuit of Figure 1.

Design this bandpass filter to meet the following specifications (exact values will be given in class):

- Center frequency of $f_0$
- Quality factor of $Q$
- Gain at center frequency $T_p$

These equations will be useful during your work:
\[ T(s) = \frac{V_o(s)}{V_i(s)} = \frac{sR_2C_1}{1 + s(R_1C_1 + R_2C_2 + R_2C_1) + s^2(R_1R_2C_1C_2)} \]

\[ \omega_0 = \frac{1}{\sqrt{R_1R_2C_1C_2}} \]

\[ T_p = \frac{R_2C_1}{R_1C_1 + R_2C_2 + R_2C_1} \]

If

\[ f_R = \frac{R_2}{\sqrt{R_1}} \]

and

\[ f_C = \frac{C_2}{\sqrt{C_1}} \]

then

\[ Q^{-1} = f_R f_C + \frac{1}{f_R f_C} + \frac{f_R}{f_C} \]

\[ T_p = Q \frac{f_R}{f_C} \]

and

\[ \omega_0 = \frac{1}{R_2C_1 f_C} = \frac{1}{R_2C_2} f_R f_C \]

Note that

\[ T(j\omega) = \frac{j\omega (\omega_0 T_p)}{(\omega_0^2 - \omega^2) + j\omega \left(\frac{\omega_0}{\omega}\right)} \]

The pre-lab must include the following (see note below):

1. Bode plots (magnitude and phase) for the response using your ideal component values (i.e. without regard to available values). Show that these results meet all of the specifications EXACTLY!

2. Bode plots (magnitude and phase) for the circuit response using available component values. Demonstrate that these values provide an acceptable level of performance. It is YOUR job to MEET the specifications as closely as possible. This may require several iterations of choosing component values. You MUST use available resistor and capacitor values in your final design. You are allowed to “build” one of the listed values if that part is not available in the lab. Do not use electrolytic capacitors.

NOTE: Use LTspice or MATLAB® to produce the Bode plots. Use a frequency range from \(0.1f_0\) to \(10f_0\).

Place a copy of your Bode plots for the final design in your lab notebook. Make sure that your pre-lab assignment meets the guidelines as outlined in section1.6.

**Procedures**
1. Construct the circuit using components which are within 10% of the design values. Measure your capacitor values. YOU ARE NOT ALLOWED TO USE RESISTOR OR CAPACITOR SUBSTITUTION BOXES.

2. CENTER FREQUENCY MEASUREMENT. With the input set to a 2 V peak-to-peak sine wave, display the output waveform on the scope. Adjust the input frequency until the output amplitude peaks. Note that this is the actual center frequency of your circuit, $f_0$. Compare to the design value. Discuss any serious discrepancies with the instructor. Your lab notebook entry should look something like:

   **Procedure Part 2 Center frequency measurement**

   Adjusted signal generator to peak output.
   Peak-to-Peak voltage of input via scope: xxx.x V
   Peak-to-Peak voltage of output via scope: xxx.x V

   Measured frequency via scope: $f_0 = xxx.x$ Hz
   Design Value: xxx.x Hz
   Percent error: x.x %

   The center frequency is off only x.x %. This is due to....

3. FREQUENCY RESPONSE MEASUREMENT. Using a sine wave input, measure and plot the magnitude and phase of your circuit transfer function as the input frequency is varied. The test frequency should range from $0.1f_0$ to $10f_0$. The test frequencies should be geometrically equally spaced over the interval (Let the first test frequency be $f_0/10$. Then each successive frequency is $k$ times the previous, where $k = 100^{(\frac{1}{10})}$. Choose the number of data points $N$ as an even number to insure that $f_0$ is one of the test frequencies. Choose $N$ at least 10). You may wish to take additional measurements as well. Use the scope’s dual channel capability to simultaneously display the input and output sine waves. For each frequency:

   a. Record the amplitudes of both. The ratio is $|T(j\omega)|$. Calculate the gain in decibels. **DO NOT ASSUME THAT THE SIGNAL GENERATOR OUTPUT VOLTAGE DOES NOT CHANGE WITH FREQUENCY!**

   b. Find $\angle T(j\omega)$ by using the scope cursors to measure the time difference between peaks of the input and output waves. Record this time difference in units of scope divisions as well as seconds. The phase angle in radians is $\omega \Delta t$ (also record the angle in degrees, 360 $f\Delta t$). **BE SURE TO NOTE WHETHER THE OUTPUT IS LEADING OR LAGGING THE INPUT.** The phase angle is positive for the output leading the input and negative for the output lagging the input.

   As you take your data compare your results to those obtained in the pre-lab. PLOT YOUR DATA ON YOUR PRE-LAB MAGNITUDE AND PHASE PLOTS. Resolve any serious discrepancies.
MEASURE (AGAIN?) THE MAGNITUDE AND PHASE AT $f_0/3$, $f_0$, and $5f_0/3$. Compare this data with your pre-lab results to insure its accuracy. You will need this data to complete the lab exercises.

4. CIRCUIT BANDWIDTH AND Q MEASUREMENT. Find and record the two frequencies at which $|T(j\omega)|$ is 3dB down from its peak. Use this to estimate the circuit bandwidth $BW$ and hence the circuit $Q = f_0/BW$. Compare to the design value.

5. GAIN COMPARISON. Compare the gain $T_p$ at $f_0$ to the design value.

6. Discuss any serious discrepancies with the instructor. Remember that it is YOUR job to meet the specifications.

7. SQUARE WAVE RESPONSE MEASUREMENT. Once your circuit is operating within specs, display the response of your circuit for square wave inputs at $f_0$, $f_0/3$, and $f_0/5$ Hz. The square waves should be symmetric about zero volts (WHY?). Display both the input and output waveforms on the scope. Get a hardcopy printout and place in your notebook.

Report

Prepare a formal report for this laboratory as discussed in detailed in section 1.5. Your report must include the exercise as described below.

Exercise

Conduct a theoretical analysis of the circuit response to the $f_0/3$ square wave. Proceed as follows. Express the input waveform as a Fourier series. Use the first three non-zero terms for your analysis, i.e.

$$v_i(t) = \frac{4V_p}{\pi} \sin \left(2\pi \frac{f_0}{3} t\right) + \frac{1}{3} \sin \left(2\pi \frac{3f_0}{3} t\right) + \frac{1}{5} \sin \left(2\pi \frac{5f_0}{3} t\right) + \cdots.$$ 

Find the circuit response to each of these three sinusoids by using your experimental gain and phase plots, i.e. $|T(j\omega)|$ and $\angle T(j\omega)$ at $f_0/3$, $f_0$, and $5f_0/3$. Denote the gains as $T_1$, $T_2$, and $T_3$, respectively, and the angles as $\Theta_1$, $\Theta_2$, and $\Theta_3$, respectively. The output voltage is thus

$$v_o(t) = \frac{4V_p}{\pi} \left[ T_1 \sin \left(2\pi \frac{f_0}{3} t + \Theta_1\right) + T_2 \frac{1}{3} \sin \left(2\pi \frac{3f_0}{3} t + \Theta_2\right) + T_3 \frac{1}{5} \sin \left(2\pi \frac{5f_0}{3} t + \Theta_3\right) + \cdots \right].$$

Plot both equations for $v_i(t)$ and $v_o(t)$ using a computer tool of your choice. Compare the results to those observed in the laboratory. Also plot each individual sinusoid in the equations for $v_i(t)$ and $v_o(t)$ to show the relative contribution of each to the input waveform.

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