Investigation of Stepsize Effects on Numerical Solutions of a Differential Equation

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1. Consider the differential equation [1]
$$\dot{x} = x^2 t, \qquad (1)$$
$$x(1) = 3.$$
Solve this differential equation and demonstrate that your solution is correct.
2. Recommend a stepsize h for numerically solving (1) over the interval $[1,1.29]$ using Euler's Method. **Do not use a MATLAB ode solver – write your own code.** One approach would be to base your recommendation on the average RMS error per point between the numerical solution and the exact solution obtained in step 1. Report the execution rime of your code for the recommended h.
3. Repeat step 2 for the fourth-order Runge-Kutta Method (RK4). **Do not use a MATLAB ode solver – write your own code.** Report the execution rime of your code for the recommended h.
4. Use the MATLAB function ode45() to numerically solve (1). Report the execution rime.
5. Plot the obtained solutions from Euler's Method, RK4, and ode45() for a minimum of 100 equally spaced time values. Also, plot the exact solution on the same graph. Plot the error for the same equally spaced time values for the three solutions.
6. Compare and contrast the effectiveness of these three methods.

Prepare a technical report describing your results. Attach a listing of all code. Be sure your code contains explanatory comments. **You are cautioned that you must write your own code.**

References

[1] Frank L. Severance, *System Modeling and Simulation: An Introduction*, John Wiley & Sons, New York, 2001.