Activity 4: Vectors, Matrices, and Linear Transformations

Names: _____ Date: October 8, 2009 Score: _____

Show you work or explain your answer for each of the following. You should submit one copy for your group. Feel free to ask your instructor for advice if you need it.

1. (8 pts) Let

$$\begin{aligned} T_1(x_1, x_2) &= (2x_1, 3x_1 - 2x_2, 5x_2) \\ T_2(x_1, x_2, x_3) &= (x_1 + x_2 - x_3) \\ T_3(x_1, x_2) &= (-3x_1 - 4x_2, x_1 + x_2) \\ T_4(x_1, x_2, x_3) &= (x_3 - x_2, x_2 - 3x_1) \end{aligned}$$

   (a) Evaluate $T_1(2, 3)$.

   (b) Which make sense? Which are nonsense? Explain.

      i. $T_1(T_3(x_1, x_2))$
      ii. $T_3(T_2(x_1, x_2, x_3))$
      iii. $T_1(T_2(x_1, x_2, x_3))$
      iv. $T_4(T_1(x_1, x_2, x_3))$

   (c) The composition of linear transformations is a linear transformation. For each composition in (b) which makes sense, find the matrix that represents the linear transformation.

2. (5 pts) Is the linear transformation

$$T(x_1, x_2, x_3) = (2x_1 + x_2 + 4x_3, -x_2 + x_3, 2x_1 + 3x_2 - x_3)$$

invertible? If so, find the inverse.

3. (7 pts)

   (a) Write a linear transformation T that rotates $\mathbb{R}^3$ clockwise around the x-axis by $\pi/2$ radians.

   (b) What is T^2? Write out a formula for the function T^2 and explain what it does geometricly.

   (c) Write a linear transformation that reflects $\mathbb{R}^3$ across the yz-plane.

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4. (bonus) Find an affine transformation that sends $(1, 1)$ to $(3, 1)$, $(2, 0)$ to $(4, -6)$, and $(0, 1)$ to $(1, 4)$.