

Activity 3: Looking back on 2.2 and before, looking forward to 2.4

Names: _____ Date: September 24, 2009 Score: _____

Show your work for each of the following. You should submit one copy for your group. Feel free to ask your instructor for advice if you need it.

1. (4 pts) For each of the following differential equations, find all equilibrium solutions. Explain both why the solutions you claim are equilibrium solutions are indeed equilibrium solutions and why there are no other equilibrium solutions.

(a) $\frac{dy}{dx} = (y - 3)(y + 2)$

(b) $\frac{dy}{dx} = (x - 3)(x + 2)$

2. Recall that for a function $f(x)$, you can approximate $f(x)$ near a with the tangent line to f at a . The tangent line has slope $f'(a)$ and goes through the point $(a, f(a))$ so the equation of the line is $y - f(a) = f'(a)(x - a)$. In other words, $f(x) \approx f(a) + f'(a)(x - a)$. We can look at approximate solutions to differential equations using this idea. Let's look at the differential equation $\frac{dy}{dx} = 3x - 5y$ with initial condition $y(1) = 2$.

- (a) (2 pts) Find the slope of the tangent line to the solution to the differential equation with initial condition $y(1) = 2$ at the point $(1, 2)$.

- (b) (2 pts) Find the equation for the line tangent to the solution of the differential equation at $(1, 2)$.

- (c) (2 pts) Use the equation for that line to approximate $y(1.1)$ for the solution to the differential equation with $y(1) = 2$. Call this approximate value c .
- (d) (2 pts) Now you have an approximate value for $y(1.1)$ for the solution to the differential equation with $y(1) = 2$, use it to approximate the slope of the tangent line to that solution when $x = 1.1$.
- (e) (2 pts) Find the equation of the line with that slope that goes through the point $(1.1, c)$.
- (f) (2 pts) Use that line to approximate $y(1.2)$.
- (g) (4 pts) You can solve the differential equation exactly. Do so, and compare your exact solutions for $y(1.1)$ and $y(1.2)$ with your approximate ones.