Some Skills from Calculus to Review

Math 272 relies heavily on your knowledge of material from your previous math courses, especially precalculus, calculus, and geometry. Here are some problems to help you review those topics.

1. Calculate derivatives and integrals: Take derivatives of e^{4x^2}, $\frac{\ln(1+x^4)}{x^2+3}$, and $sin(x-2)+x^3-5^x$ and integrate x^2+3x-2, $\frac{1}{x}$, $\frac{x+4}{x^2+4x-5}$, $\cos(x)e^{\sin(x)}$, and $x\ln(x)$.

2. Derivative as the slope of a tangent line: Consider the function $f(x)=\ln(2+\sin(x))$. What is the derivative of f? What is the derivative of f at $x=\pi$? What is the equation of the tangent line to f at $x=\pi$? Use this equation to approximate f when $x=\pi-.01$.

3. Integral is sometimes area: The value of some definite integrals calculate an area between the graph of a function and the x axis, others should be interpreted as the difference of areas. How do the following integrals relate to areas? Make sure you can explain this with a picture involving the graph of the function.

   (a) $\int_0^{10} x^3\,dx$
   (b) $\int_0^{\pi} \sin(x)\,dx$
   (c) $\int_{-5}^{5} x\,dx$

4. Approximating functions by Taylor series: If $f(x)=e^{3x+2}$, find a polynomial that , at $x=5$, has the same value, first derivative, and second derivative as f does. Can you find Taylor series that agrees with f at $x=5$ for all derivatives?

5. Existence of inverse functions: Which of these functions are invertible? If they are not invertible, is there a way to restrict their domain so that the function restricted to the smaller domain is invertible?

   (a) e^{3x-2}
   (b) $x+3$
   (c) x^2+3
   (d) $sin(x)$

6. Implicitly defined functions, and when they have an inverse: Consider the graph of solutions to the equations $x^2+5y^2=5$ or $x^2+y^3=5$. Is y a function of x? Is it possible to pick a domain for x so that y is a function of x? How about picking a domain for y so that x is a function of y?

7. Equations of circles, ellipses, parabolas, and hyperbolas: Graph solutions to the following equations without use of a calculator:

   (a) $\frac{(x-3)^2}{4}+\frac{(y+5)^2}{9}=25$
   (b) $\frac{(x-3)^2}{4}-\frac{(y+5)^2}{9}=25$
   (c) $-\frac{(x-3)^2}{4}+\frac{(y+5)^2}{9}=25$
   (d) $\frac{(x-3)^2}{4}-\frac{(y+5)}{9}=25$

8. Distance formula/trig: If a clock has a minute hand which is 4 inches long and an hour hand which is 3 inches long, how far apart are the tips of the hands at noon? How about at 12:15? How about 2:34? One method: Find the the locations of the tips of the hands in $\mathbb{R}^2$ and taking the distance between them. Another method: draw an appropriate triangle, find two sides and one angle,then use trig to find the other side.