

Chapter 10

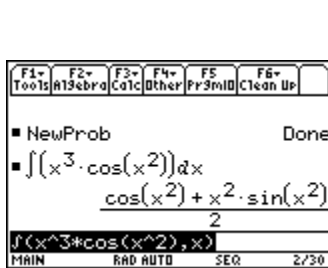
Techniques of Integration

- §1. Symbolic Checking of Other Techniques
- §2. Calculator Substitutions
- §3. Algebra with Rational Functions

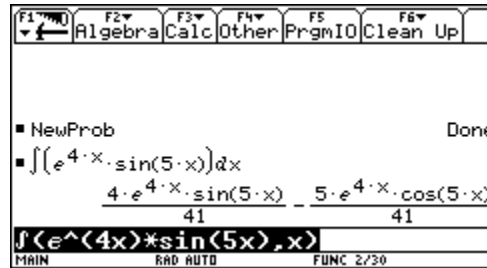
The activities in this chapter present symbolic activities involving integrals. This corresponds to Chapter 10 of the text *Calculus with Early Vectors*, by Phillip Zenor, Edward Slaminka, and Donald Thaxton, Prentice Hall, 1999.

1. Symbolic Checking of Other Techniques

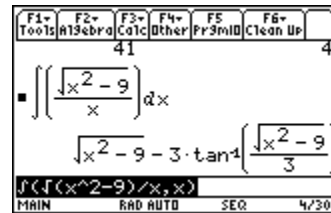
The TI-89 can handle most of the integrals that you might be able to evaluate using integration by parts or substitution. The calculator will completely evaluate the integral while your technique may only transform the integral to a simpler one. Still you can often check your hand work by having the calculator do the same problem. Remember that the calculator may get a different antiderivative (which may differ by a constant) and that the answer may be in a very different algebraic form.



Integration by parts



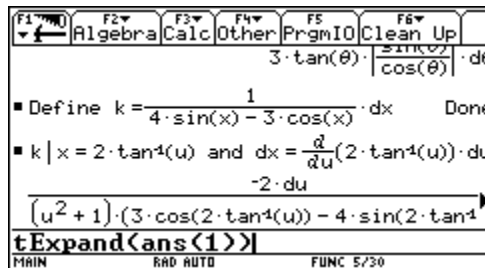
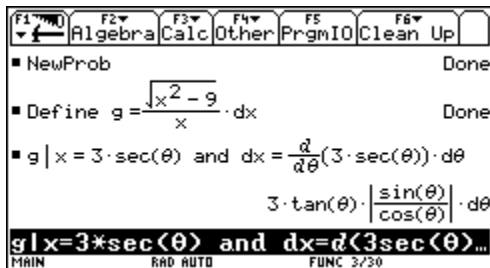
Integration by parts twice



Substitute $x = 3 \sec \theta$

2. Calculator Substitutions

You may be able to get the calculator to do some of the algebraic work involved in completing a substitution.



$$\frac{(u^2+1) \cdot (3 \cdot \cos(2 \cdot \tan^4(u)) - 4 \cdot \sin(2 \cdot \tan^4(u)))}{3 \cdot u^2 + 8 \cdot u - 3}$$

$$\int \frac{(u^2+1) \cdot (3 \cdot \cos(2 \cdot \tan^4(u)) - 4 \cdot \sin(2 \cdot \tan^4(u)))}{3 \cdot u^2 + 8 \cdot u - 3} \cdot 2 \cdot du$$

tExpand(Ans(1))

$$\int \frac{1}{4 \cdot \sin(x) - 3 \cdot \cos(x)} dx$$

$$-\ln\left(\frac{3 \cdot \cos(x) + \sin(x) + 3}{-\cos(x) + 3 \cdot \sin(x) - 1}\right) + C$$

f(1/(4*sin(x)-3*cos(x)),x)

3. Algebra with Rational Functions

When integrating a rational function, the first step is to perform “long division” of the numerator by the denominator to convert the task into a problem of integrating a polynomial plus a rational function with the degree of the numerator strictly less than the denominator. Then you use partial fractions on the last piece. The F2 Algebra commands 3:expand(and 7:propFrac(may do some this algebraic work.

$$\frac{5 \cdot x^4 + 2 \cdot x}{x^3 - 6 \cdot x^2 + 3 \cdot x + 10}$$

$$\frac{1}{6 \cdot (x+1)} - \frac{28}{3 \cdot (x-2)} + \frac{1045}{6 \cdot (x-5)} + 5 \cdot x + 30$$

expand((5x^4+2x)/(x^3-6x^2+3x+10))

$$\frac{5 \cdot x^4 + 2 \cdot x}{x^3 - 6 \cdot x^2 + 3 \cdot x + 10}$$

$$\frac{3 \cdot (55 \cdot x^2 - 46 \cdot x - 100)}{x^3 - 6 \cdot x^2 + 3 \cdot x + 10} + 5 \cdot x + 30$$

propFrac((5x^4+2x)/(x^3-6x^2+3x+10))

$$\frac{3 \cdot (55 \cdot x^2 - 46 \cdot x - 100)}{x^3 - 6 \cdot x^2 + 3 \cdot x + 10} + 5 \cdot x + 30$$

$$\int \left(\frac{5 \cdot x^4 + 2 \cdot x}{x^3 - 6 \cdot x^2 + 3 \cdot x + 10} \right) dx$$

$$\frac{\ln(x+1)}{6} - \frac{28 \cdot \ln(x-2)}{3} + \frac{1045 \cdot \ln(x-5)}{6} + 5x + 30$$

f(5x^4+2x)/(x^3-6x^2+3x+10),x)

$$\frac{3 \cdot (55 \cdot x^2 - 46 \cdot x - 100)}{x^3 - 6 \cdot x^2 + 3 \cdot x + 10} + 5 \cdot x + 30$$

$$\int \left(\frac{5 \cdot x^4 + 2 \cdot x}{x^3 - 6 \cdot x^2 + 3 \cdot x + 10} \right) dx$$

$$\frac{3 \cdot \ln(x-2)}{3} + \frac{1045 \cdot \ln(x-5)}{6} + \frac{5 \cdot x^2}{2} + 30 \cdot x$$

f(5x^4+2x)/(x^3-6x^2+3x+10),x)

$$\frac{4 \cdot x}{(x^2+1)^3}$$

$$\frac{4 \cdot x^2}{(x^2+1)^3}$$

$$\frac{4}{(x^2+1)^2} - \frac{4}{(x^2+1)^3}$$

expand(4x^4/((x^2+1)^3))

$$\frac{4 \cdot x^4}{(x^2+1)^3}$$

$$\frac{4}{x^2+1} - \frac{8}{(x^2+1)^2} + \frac{4}{(x^2+1)^3}$$

expand(4x^4/((x^2+1)^3))

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
$\frac{x^2+1}{(x^2+1)^2} + \frac{1}{(x^2+1)^3}$					
$\blacksquare \text{ expand } \left(\frac{x^3+x^2+x+1}{(x^2+9)^2} \right)$					
$\frac{x}{x^2+9} + \frac{1}{x^2+9} - \frac{8 \cdot x}{(x^2+9)^2} - \frac{8}{(x^2+9)^2}$					
$\text{d} \left(\frac{x^3+x^2+x+1}{(x^2+9)^2} \right)$					
MAIN	RAD AUTO		FUNC 4/30		

F1	F2	F3	F4	F5	F6
Algebra	Calc	Other	PrgmIO	Clean Up	
$\frac{x^2+9}{x^2+9} + \frac{1}{x^2+9} + \frac{1}{(x^2+9)^2} + \frac{1}{(x^2+9)^3}$					
$\blacksquare \int \left(\frac{x^3+x^2+x+1}{(x^2+9)^2} \right) dx$					
$\frac{\ln(x^2+9)}{2} + \frac{5 \cdot \tan^{-1}\left(\frac{x}{3}\right)}{27} - \frac{4 \cdot (x-9)}{9 \cdot (x^2+9)}$					
$\int \left(\frac{x^3+x^2+x+1}{(x^2+9)^2} \right) dx, x \dots$					
MAIN	RAD AUTO		FUNC 5/30		

Since the calculator can do symbolic integration so easily, there seems to be no need for partial fractions and then hand integration of the pieces. However this technique of expanding a rational function into its partial fraction decomposition has other uses besides integration, and you should learn about it. For example, electrical engineers using the Laplace transform will find the technique of partial fractions useful for finding inverse transforms.