1. Solve the following inequalities for the acute angle x:

(a) $1/2 - \sin x \geq 0$;

(b) $\sqrt{3}/2 - \cos x \leq 0$;

(c) $\sqrt{3}/2 - \cot x > 0$;

(d) $\sqrt{3} - \tan x \geq 0$;

2. Solve the following inequalities for the angle x such that $\pi \leq x \leq 2\pi$:

(a) $\sqrt{2}/2 + \sin x \geq 0$;

(b) $1/2 + \cos x < 0$;

(c) $\tan x - \sqrt{3}/3 \leq 0$;

(d) $\cot x - \sqrt{3} < 0$;

3. Calculate $\sin\alpha$ and $\cos\alpha$ if:

(a) $3\sin\alpha + 4\cos\alpha = 5$;

(b) $3\sin\alpha - 3\sin^2\alpha + \cos^2\alpha - 1 = 0$.

4. Write the following expressions in terms of: (1) $\tan\alpha$; (2) $\cot\alpha$.

(a) $\dfrac{3\sin\alpha\cos\alpha}{4\sin^2\alpha + 3\cos^2\alpha}$;

(b) $\dfrac{\sin\alpha\cos\alpha}{5\sin^2\alpha + 3\cos^2\alpha}$;

(c) $\dfrac{2\sin\alpha - 3\sin^3\alpha}{5\cos^2\alpha - 4\cos\alpha}$;

5. Simplify the following expressions:

(a) $\left(\dfrac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha} + \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right)\left(\dfrac{1}{\sin^2\alpha} + \dfrac{1}{\cos^2\alpha}\right)$;

(b) $\left(\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} - \sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}\right)^2$;

(c) $\dfrac{(\sin\alpha + \cos\alpha)^2 - (\sin\alpha - \cos\alpha)^2}{\cos\alpha\sin\alpha}$;

(d) $\dfrac{\sin(\alpha+\beta) - \cos\alpha\sin\beta}{\sin(\alpha+\beta) - \sin\alpha\cos\beta}$;

(e) $\dfrac{\sin(\alpha-\beta) + \cos\alpha\sin\beta}{\cos(\alpha-\beta) - \sin\alpha\sin\beta}$;

(f) $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)}$;

(g) $(\sin\alpha+\cos\beta)^2+(\cos\alpha+\sin\beta)^2-2\sin(\alpha+\beta)$;

(h) $(\sin\alpha-\sin\beta)^2+(\cos\alpha-\cos\beta)^2+2\cos(\alpha-\beta)$;

(i) $\dfrac{\sin^3\alpha-\cos^3\beta}{\sin\alpha-\cos\beta}+\dfrac{\cos^3\alpha-\sin^3\beta}{\cos\alpha-\sin\beta}-\sin(\alpha+\beta)$;

(j) $\dfrac{\cos^3\alpha+\cos^3\beta}{\cos\alpha+\cos\beta}+\dfrac{\sin^3\alpha+\sin^3\beta}{\sin\alpha+\sin\beta}+\cos(\alpha-\beta)$;

(k) $\dfrac{2\sin^2 x-\sin 2x}{2\cos^2 x-\sin 2x}$;

(l) $\dfrac{(\cos x-\sin x)^2-\cos 2x}{2\sin^2 x-\sin 2x}$;

(m) $\dfrac{\sin x\sin 2x}{\cos x-\cos x\cos 2x}$;

(n) $\dfrac{\cos x\sin 2x}{\sin x+\sin x\cos 2x}$.

6. Given that $\sin x+\cos x=m$, calculate $\sin x\cos x$.

7. Given that $\tan x+\cot x=m$, calculate:

(a) $\tan^2 x+\cot^2 x$.

(b) $\tan^3 x+\cot^3 x$.

8. Prove the following identities:

(a) $\dfrac{\sin\alpha}{\sin\alpha+\cos\alpha}-\dfrac{\cos\alpha}{\cos\alpha-\sin\alpha}=\dfrac{\tan^2\alpha+1}{\tan^2\alpha-1}$;

(b) $\cos^2\alpha+2\sin^2\alpha+\sin^2\alpha\tan^2\alpha=1/\cos^2\alpha$;

(c) $\cos^2\alpha(\tan\alpha+2)(2\tan\alpha+1)-5\sin\alpha\cos\alpha=2$;

(d) $\cos\alpha(1-\tan\alpha)(\sin\alpha+\cos\alpha)=\cos^4\alpha-\sin^4\alpha$;

(e) $\dfrac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}=\dfrac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$;

(f) $\cos^2 x-2\cos x\cos\alpha\cos(\alpha+x)+\cos^2(\alpha+x)=\sin^2 x$;

(g) $\dfrac{\cos x-\sin x}{\cos x+\sin x}=\dfrac{\cos 2x}{1+\sin 2x}$;

(h) $\tan(\pi/4 + x) - \tan(\pi/4 - x) = 2\tan 2x$;

(i) $\tan(\pi/6 + x)\tan(\pi/6 - x) = \dfrac{2\cos 2x - 1}{2\cos 2x + 1}$;

(j) $\tan\alpha + 2\tan 2\alpha = \cot\alpha - 4\cot 4\alpha$;

(k) $\sin 18^\circ \cos 36^\circ = 1/4$;

(l) $\tan(x/2) + 2\sin^2(x/2)\cot x = \sin x$;

(m) $\sin(30^\circ + \alpha) + \sin(30^\circ - \alpha) = \cos\alpha$;

(n) $\cos(60^\circ - \alpha) + \cos(60^\circ + \alpha) = \cos\alpha$;

(o) $\sin(45^\circ + \alpha) + \sin(45^\circ - \alpha) = \sqrt{2}\cos\alpha$;

(p) $\cos(60^\circ - \alpha) - \cos(60^\circ + \alpha) = \sqrt{3}\sin\alpha$;

(q) $\dfrac{\cos 2x - \cos 4x}{\cos 2x + \cos 4x} = \tan 3x \tan x$.

9. Without using calculators evaluate:

(a) $\sin(\pi/3 + \alpha)$, if $\cos\alpha = 1/2$ and $0 \le \alpha \le \pi/2$.

(b) $\cos(2\pi/3 - \alpha)$, if $\sin\alpha = \sqrt{3}/2$ and $\pi/2 \le \alpha \le \pi$.

(c) $\sin(\alpha + \beta)$, if $\sin\alpha = 1/3$, $\cos\beta = -\sqrt{3}/2$, and $0 \le \alpha \le \pi/2$, $\pi/2 \le \beta \le \pi$.

(d) $\tan(\pi/4 + \alpha)$, if $\cos\alpha = 1/3$ and $0 \le \alpha \le \pi/2$.

(e) $\tan(\alpha - \beta)$, if $\sin\alpha = 2/3$, $\cos\beta = 1/2$, and $0 \le \alpha \le \pi/2$, $0 \le \beta \le \pi/2$.

(f) $\cot(3\pi/4 - \alpha)$, if $\cos\alpha = 4/5$ and $3\pi/2 \le \alpha \le 2\pi$;

(g) $\sin 2\alpha$, if $\sin\alpha = 1/2$ and $0 \le \alpha \le \pi/2$;

(h) $\cos 2\alpha$, if $\cos\alpha = -\sqrt{3}/2$ and $\pi/2 \le \alpha \le \pi$.

(i) $\tan 2\alpha$, if $\sin\alpha = 1/2$ and $0 \le \alpha \le \pi/2$;

(j) $\cos\beta$, if $\cos\alpha = 1/7$, $\cos(\alpha + \beta) = -11/14$, and $0 \le \alpha \le \pi/2$, $0 \le \beta \le \pi/2$;

(k) $\sin(\alpha - \beta + \gamma)$, if $\sin\alpha = 2/3$, $\sin\beta = 3/4$, $\sin\gamma = 4/5$, and α, β, γ are acute angles;

(l) $12\sin^2\alpha - 7\sin 2\alpha + 4\cos^2\alpha$, if $\tan\alpha = 2/3$;

(m) $\sin 3\alpha$ and $\cos 3\alpha$, if $\sin\alpha = 1/3$.

10. If α, β, γ are the angles of a triangle and $\sin^2\alpha + \sin^2\beta = \sin^2\gamma$, show that this triangle is a right triangle.

11. If $\alpha+\beta+\gamma=\pi$ show that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta\cos\gamma=1$.

12. Given that $\sin x-\cos x=m$, calculate $\sin 2x$.

13. Without using calculators evaluate $\sin\alpha$, $\cos\alpha$, and $\tan\alpha$ if:

(a) $\sin 2\alpha=\sqrt{3}/2$, $0\le 2\alpha\le\pi/2$;

(b) $\cos 2\alpha=-1/2$, $\pi/2\le 2\alpha\le\pi$;

(c) $\tan 2\alpha=\sqrt{3}$, $0\le 2\alpha\le\pi/2$;

(d) $\cot 2\alpha=-\sqrt{3}/3$, $0\le 2\alpha\le\pi$.

14. Calculate $\sin 2\alpha$, $\cos 2\alpha$, and $\tan 2\alpha$ if:

(a) $\sin\alpha=2a/(a^2+1)$;

(b) $\cos\alpha=(a^2-b^2)(a^2+b^2)$;

(c) $\tan\alpha=2ab/(a^2-b^2)$.

15. Reduce the fractions:

(a) $\dfrac{\sin 3x}{\sin 6x}$;

(b) $\dfrac{\cos x}{\sin 2x}$;

(c) $\dfrac{\cos x}{1+\cos 2x}$;

(d) $\dfrac{1-\cos x}{\sin(x/2)}$;

(e) $\dfrac{1+\cos x}{\sin x}$;

(f) $\dfrac{1+\cos 2x}{\cos x}$.

16. If $3\tan\alpha=2\tan\beta$ then $\tan(\beta-\alpha)=\dfrac{\sin 2\alpha}{5-\cos 2\alpha}$.

17. Without using calculators evaluate $\sin(\pi/8)$, $\cos(\pi/8)$, and $\tan(\pi/8)$.

18. Show that:

(a) $\sin\dfrac{\pi}{16}=\dfrac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2}}}$;

(b) $\cos\dfrac{\pi}{16}=\dfrac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2}}}$;

(c) $\cos^2 \dfrac{\pi}{32} = \dfrac{2 - \sqrt{2 + \sqrt{2 + \sqrt{2}}}}{4}$.

19. If $\cos\alpha = 40/41$, $\cos\beta = 60/61$, and α, β are acute angles, then $\sin^2 \dfrac{\alpha - \beta}{2} = \dfrac{1}{41 \cdot 61}$.

20. If $\alpha + \beta + \gamma = \pi$ then $\sin\alpha + \sin\beta + \sin\gamma = 4\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2}\cos\dfrac{\gamma}{2}$.

21. Without using calculators verify the following equalities:

(a) $\sin 40^\circ + \sin 20^\circ = \cos 10^\circ$;

(b) $\sin 40^\circ + \cos 70^\circ = \sin 80^\circ$;

(c) $\cos 75^\circ - \cos 15^\circ = -\sqrt{2}/2$;

22. Factor the following expressions:

(a) $1 + \sin x$;

(b) $1 + \cos x$;

(c) $1 - \sin x$;

(d) $1 - \cos x$;

(e) $1/2 + \sin x$;

(f) $1/2 - \sin x$;

(g) $\sqrt{3}/2 + \cos x$;

(h) $\sqrt{2}/2 + \sin x$;