

1. Prove that $\lim_{n \rightarrow \infty} \frac{1}{1 + 3n} = 0$.
2. Prove that $\lim_{n \rightarrow \infty} \frac{6n^2 + 5}{2n^2 - 3n} = 3$.
3. Prove that the sequence $a_n = 1 + (-1)^n$ does not converge.
4. Prove or disprove: If $\lim a_n = L$ then $\lim |a_n| = |L|$.
5. Give an example of a convergent sequence of rational numbers that has an irrational limit.
6. Give an example of a convergent sequence of irrational numbers that has a rational limit.
7. Prove or disprove: If a_n and b_n are divergent sequences then so is $a_n + b_n$.
8. Prove or disprove: If a_n and b_n are divergent sequences then so is $a_n b_n$.
9. Give an example of a convergent sequence a_n of positive numbers so that $\lim(a_{n+1}/a_n) = 1$.
10. Give an example of a divergent sequence a_n of positive numbers so that $\lim(a_{n+1}/a_n) = 1$.
11. Prove that the sequence given by $a_1 = 1$ and $a_{n+1} = (a_n + 5)/4$ is monotone and bounded. Then find its limit.

12. Prove or disprove: If a convergent sequence is bounded then it is monotone.

13. Prove or disprove: If a convergent sequence is monotone then it is bounded.

14. Find the supremum for each of the sets:

(a) $\{0, 1\}$;

(b) $[0, 1]$;

(c) $\{1/n : n \in \mathbb{N}\}$.

15. Let S be a nonempty subset of \mathbb{R} and let $c \in \mathbb{R}$. Define $cS = \{cx : x \in S\}$. Prove that if $k \geq 0$ then $\sup kS = k \sup S$.