

# ON THE SIMILARITY OF CENTERED OPERATORS TO CONTRACTIONS

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ABSTRACT. In this paper we show that every power bounded operator weighted shift with commuting normal weights is similar to a contraction. As an application of this result, we reduce the problem whether every centered power bounded operator is similar to a contraction to the analogous question about quasi-invertible centered operators.

## 1. INTRODUCTION

Let  $\mathcal{H}$  be a separable, infinite dimensional, complex Hilbert space, and let  $\mathcal{L}(\mathcal{H})$  denote the algebra of all bounded linear operators acting on  $\mathcal{H}$ . An operator  $T$  in  $\mathcal{L}(\mathcal{H})$  is said to be *power bounded* if there exists an  $M > 0$  such that for every positive integer  $n$ ,  $\|T^n\| \leq M$ , and to be *polynomially bounded* if there exists a constant  $K > 0$  such that

$$\|p(T)\| \leq K \sup\{|p(\xi)| : |\xi| = 1\}$$

for every polynomial  $p$ . For simplicity of reference, we shall denote the class of all power bounded operators in  $\mathcal{L}(\mathcal{H})$  by  $PW(\mathcal{H})$  and the class of all polynomially bounded operators in  $\mathcal{L}(\mathcal{H})$  by  $PB(\mathcal{H})$ . An operator  $T$  in  $\mathcal{L}(\mathcal{H})$  is said to be similar to a contraction (notation:  $T \in \mathcal{SC}(\mathcal{H})$ ) if there exists an invertible operator  $X$  in  $\mathcal{L}(\mathcal{H})$  such that  $\|XTX^{-1}\| \leq 1$ .

For a long time it was an open question whether  $PB(\mathcal{H}) \subset \mathcal{SC}(\mathcal{H})$ , but eventually Foguel [1] provided a counterexample, which Halmos simplified in [2]. Further analysis of this example inspired the latter to formulate explicitly in [3] the question whether

$$(1) \quad PB(\mathcal{H}) \subset \mathcal{SC}(\mathcal{H}).$$

Of course, it is obvious from the definition and von Neumann's inequality for contraction operators [14] that  $\mathcal{SC}(\mathcal{H}) \subset PB(\mathcal{H})$ . Several authors have addressed the question whether (1) is valid (cf. [4], [5], [6], [10]), and we mention, in particular, some nice progress made by Paulsen [8], but, as of this writing, the question remains open. An affirmative answer to this problem would produce an instantaneous structure theory for the class  $PB(\mathcal{H})$  – namely, the one obtained by transporting the much studied structure theory for contraction operators (cf. [14]) via similarity transformations to the class  $PB(\mathcal{H})$ . Furthermore, an affirmative answer is known for some classes of operators including the compact operators ([13]) and the scalar-weighted shifts ([11]).

An operator  $T \in \mathcal{L}(\mathcal{H})$  is said to be *weakly centered* if  $TT^*$  commutes with  $T^*T$ . An operator  $T \in \mathcal{L}(\mathcal{H})$  is said to be *centered* if the doubly infinite sequence

$$\dots, T^2T^{*2}, TT^*, T^*T, T^{*2}T^2, \dots$$

consists of mutually commuting operators. Centered operators have been studied in [7]. If we write the polar decomposition  $T = UB$  of a centered operator, then it is possible (cf.

[7]) to classify centered operators by considering the structure of the partial isometry  $U$  in the decomposition above. Namely, a centered operator is of type I if  $U$  is a pure isometry, of type II if  $U$  is a pure coisometry, of type III if  $U$  is a direct sum of nilpotent power partial isometries, and of type IV if  $U$  is unitary.

The pertinence of weakly centered and centered operators to the question whether (1) holds arises from the fact that in [10] the author reduced the question (1) to the question whether every weakly centered, polynomially bounded operator is similar to a contraction. Since centered operators are the best known examples of weakly centered operators, this led naturally to the question whether every centered polynomially bounded operator is similar to a contraction. It is the purpose of this paper to show that every power bounded centered operator of type I, II, or III is similar to a contraction, and thus that every centered operator in  $PW$  belongs to  $SC$  if and only if every such operator of type IV does.

Before stating the main results of this paper we briefly mention some notation and terminology. As usual,  $\mathbb{N}$  is the set of positive integers,  $\mathbb{N}_0$  is the set of nonnegative integers and  $\mathbb{Z}$  denotes the set of all integers. If  $\{A_n\}_{n \in \mathbb{Z}}$  is a sequence of operators in  $\mathcal{L}(\mathcal{H})$  we denote by  $\text{diag}(\{A_n\}_{n=-\infty}^{\infty})$  the direct sum  $\bigoplus_{n=-\infty}^{\infty} A_n$  acting on  $\mathcal{H}(\mathbb{Z})$ , the direct sum of  $\mathbb{N}_0$  copies of  $\mathcal{H}$  indexed by  $\mathbb{Z}$ , in the obvious fashion. As usual  $C(X)$  is the algebra of all continuous complex-valued functions on  $X$ . A topological space  $X$  is said to be extremally disconnected if  $X$  has a topological base consisting of compact open sets and the closure of every open set is open.

**THEOREM 1.1.** *Every centered operator of type I, II, or III, as well as any direct sum of such operators, is similar to a contraction if and only if it is power bounded.*

The proof of Theorem 1.1 depends on an analysis of several cases. By a theorem of Morrel and Muhly [7], every centered operator of type I is unitarily equivalent to a forward, unilateral, operator-weighted shift whose weights are mutually commuting, Hermitian, quasi-invertible operators. Theorem 1.1 asserts that such an operator is similar to a contraction if and only if it is power bounded. However, more is true.

**THEOREM 1.2.** *If  $T \in \mathcal{L}(\mathcal{H})$  is a (forward, backward, or bilateral) operator-weighted shift with commuting normal operators as weights, then  $T \in SC(\mathcal{H})$  if and only if  $T \in PW(\mathcal{H})$ .*

## 2. PROOF OF THEOREM 1.2

*Proof.* Since the inclusion  $SC(\mathcal{H}) \subset PB(\mathcal{H})$  is well known, we concentrate on the opposite inclusion. We will give the proof in the case when  $T$  is a bilateral (forward) operator weighted shift. The case when  $T$  is a forward, unilateral, operator weighted shift can be expounded in a similar way, and the case when  $T$  is a backward shift (unilateral or bilateral) follows from the facts that a backward shift is the adjoint of a forward one, and that the classes  $PW$  and  $SC$  are selfadjoint.

Given a bilateral (forward) operator weighted shift  $T$  with weight sequence  $\{A_i\}_{i \in \mathbb{Z}}$  consisting of mutually commuting normal operators, let  $\mathcal{A}$  be the (abelian) von Neumann algebra generated by the sequence  $\{A_i\}_{i \in \mathbb{Z}}$ . Then one knows (see [12]) that there exists an extremally disconnected, compact Hausdorff space  $X$  and an isometric  $*$ -isomorphism  $\rho$  of  $C(X)$  onto  $\mathcal{A}$ . Let  $f_n = \rho^{-1}(A_n)$ ,  $n \in \mathbb{Z}$ . Also, there exists a decomposition  $\mathcal{H} = \sum_{n=-\infty}^{\infty} \bigoplus \mathcal{H}_n$  of the Hilbert space  $\mathcal{H}$  such that  $A_i(\mathcal{H}_i) \subset \mathcal{H}_{i+1}$ . We will construct a bounded invertible operator  $S$  of the form

$$(2) \quad \text{diag}(\{s_n\}_{n=-\infty}^{\infty})$$

relative to the same decomposition of  $\mathcal{H}$  (where the sequence  $\{g_n\} \subset C(X)$  is to be constructed), such that  $S^{-1}TS$  is a contraction.

First we define functions  $\{\tilde{f}_n\}_{n \in \mathbb{Z}}$  in  $C(X)$  inductively. Let  $x \in X$ . We define

$$\tilde{f}_0(x) = \max\{|f_0(x)|, 1\},$$

and notice that  $\tilde{f}_0(x) \neq 0$  for  $x \in X$ . If  $n \in \mathbb{N}$  and if  $\tilde{f}_0(x), \tilde{f}_1(x), \dots, \tilde{f}_{n-1}(x)$  are defined and invertible, then for each  $x \in X$  we set

$$p_{i,j}(x) = \begin{cases} \left| \prod_{k=i}^j f_k(x) \right|, & i \leq j, \\ 1, & \text{otherwise,} \end{cases}$$

$$\tilde{p}_{i,j}(x) = \begin{cases} \prod_{k=i}^j \tilde{f}_k(x), & i \leq j, \\ 1, & \text{otherwise,} \end{cases}$$

$$q_n(x) = \min\left\{1, \min_{0 \leq i \leq n-1} \frac{1}{\tilde{p}_{i,n-1}(x)}\right\},$$

$$r_n(x) = \min\left\{1, \min_{n+1 \leq j \leq -1} \frac{1}{\tilde{p}_{n+1,j}(x)}\right\},$$

and

$$\tilde{f}_n(x) = \max\{|f_n(x)|, q_n(x)\}.$$

Next, we define

$$\tilde{f}_{-1}(x) = \max\{|f_{-1}(x)|, 1\},$$

and, as before, if  $n$  is a negative integer different from  $-1$  and if  $\tilde{f}_{-1}(x), \tilde{f}_{-2}(x), \dots, \tilde{f}_{n+1}(x)$  are defined and invertible, then for each  $x \in X$  we set

$$\tilde{f}_n(x) = \max\{|f_n(x)|, r_n(x)\}.$$

We see that each  $\tilde{f}_n$  is a positive invertible function in  $C(X)$ , and moreover that the bilateral operator weighted shift  $\tilde{T}$  with weight sequence  $\{\rho(\tilde{f}_n)\}_{n \in \mathbb{Z}}$  is power bounded. Indeed, we shall verify that for all  $m, n \in \mathbb{Z}$  with  $m \leq n$ ,

$$\left\| \prod_{k=m}^n \rho(\tilde{f}_k) \right\| \leq M^2.$$

Let  $J$  be the set of all integers  $k$  that satisfy  $m \leq k \leq n$ , and let  $J_+$  (respectively,  $J_-$ ) be the subsets of  $J$  consisting of its nonnegative (resp. negative) elements. The expression on the left-hand side of the last inequality equals

$$\left\| \prod_{k \in J_-} \rho(\tilde{f}_k) \prod_{k \in J_+} \rho(\tilde{f}_k) \right\| = \left\| \rho \left( \prod_{k \in J_-} \tilde{f}_k \prod_{k \in J_+} \tilde{f}_k \right) \right\|_{C(X)} \leq \sup_{x \in X} \left| \prod_{k \in J_-} \tilde{f}_k(x) \right| \sup_{x \in X} \left| \prod_{k \in J_+} \tilde{f}_k(x) \right|.$$

Now we show that for each  $x$  in  $X$  both

$$(3) \quad \left| \prod_{k \in J} \tilde{f}_k(x) \right| \leq M.$$

and

$$(4) \quad \left| \prod_{k \in J_+} \tilde{f}_k(x) \right| \leq M.$$

In order to do that, we will show that for each  $x$  in  $X$

$$(3') \quad \tilde{p}_{m,n}(x) \leq \max\left\{1, \max_{m \leq j \leq n} p_{m,j}(x)\right\}$$

for any  $m \leq n < 0$ , and that

$$(4') \quad \tilde{p}_{m,n}(x) \leq \max\left\{1, \max_{m \leq i \leq n} p_{i,n}(x)\right\}$$

for any  $0 \leq m \leq n$ .

Let  $x$  in  $X$  and  $n < 0$  be fixed. We will prove (3') by induction on  $m$ . If  $m = n$ , we need to show that  $\tilde{f}_n(x) \leq \max\{1, |f_n(x)|\}$ . If  $|f_n(x)| \leq r_n(x)$ , then  $\tilde{f}_n(x) \leq 1$ . If  $|f_n(x)| > r_n(x)$ , then  $\tilde{f}_n(x) = |f_n(x)|$ . Let (3') be true for some  $m \leq n$ . We will prove that it will still be valid with  $m - 1$  instead of  $m$ . If  $|f_{m-1}(x)| \leq r_{m-1}(x)$ , then

$$\tilde{f}_n(x) = r_{m-1}(x) \leq \min_{m \leq j \leq -1} \frac{1}{\tilde{p}_{m,j}(x)} \leq \frac{1}{\tilde{p}_{m,n}(x)},$$

so that  $\tilde{p}_{m-1,n}(x) = \tilde{f}_{m-1}(x)\tilde{p}_{m,n}(x) \leq 1$ . If  $|f_{m-1}(x)| > r_{m-1}(x)$ , then  $\tilde{f}_{m-1}(x) = |f_{m-1}(x)|$ , so that

$$\begin{aligned} \tilde{p}_{m-1,n}(x) &= |f_{m-1}(x)|\tilde{p}_{m,n}(x) \leq |f_{m-1}(x)| \max\left\{1, \max_{m \leq j \leq n} p_{m,j}(x)\right\} = \\ &= \max\left\{|f_{m-1}(x)|, \max_{m \leq j \leq n} |f_{m-1}(x)|p_{m,j}(x)\right\} = \max\left\{p_{m-1,m-1}(x), \max_{m \leq j \leq n} p_{m-1,j}(x)\right\} = \\ &= \max_{m-1 \leq j \leq n} p_{m-1,j}(x). \end{aligned}$$

The inequality (4') can be proved in an analogous way, namely by induction on  $n$ . We omit the details. The power boundedness of  $T$  implies that the right hand sides of (3') and (4') are bounded by  $M$ , which shows that (3) and (4) are true. We conclude that  $\tilde{T}$  is in the class  $PW$ .

Now we define the functions  $\{g_n\}$  for each  $n \in \mathbb{Z}$ :

$$g_n(x) = \begin{cases} \frac{\tilde{p}_{0,n-1}(x)}{\min_{0 \leq k \leq n} \tilde{p}_{0,k-1}(x)}, & \text{if } n \in \mathbb{N}_0, \\ \frac{\min_{n \leq k \leq 0} \tilde{p}_{k,-1}(x)}{\tilde{p}_{n,-1}(x)}, & \text{if } n = -1, -2, \dots \end{cases}$$

It is obvious that each  $g_n$  is in  $C(X)$ . Therefore, for each  $n$  in  $\mathbb{Z}$ , the operator  $\rho(g_n)$  is well defined. Furthermore, for  $n \in \mathbb{N}$ ,

$$\|\rho(g_n)\| = \|g_n\| = \left\| \frac{\tilde{p}_{0,n-1}(x)}{\min_{0 \leq k \leq n} \tilde{p}_{0,k-1}(x)} \right\| = \left\| \frac{\tilde{p}_{0,n-1}(x)}{\tilde{p}_{0,k-1}(x)} \right\|$$

for some  $k$  satisfying  $0 \leq k \leq n$ , and therefore

$$\|\rho(g_n)\| = \left\| \frac{\tilde{p}_{0,n-1}(x)}{\tilde{p}_{0,k-1}(x)} \right\| \leq M$$

On the other hand, for  $n = -1, -2, \dots$ ,

$$\|\rho(g_n)\| = \|g_n\| = \left\| \frac{\min_{n \leq k \leq 0} \tilde{p}_{k,-1}(x)}{\tilde{p}_{n,-1}(x)} \right\| \leq 1,$$

and since, obviously,  $g_0(x) = 1$  it follows that the operator  $S$  defined in (2) is a bounded linear operator.

Moreover, it is invertible. To see that, let a sequence  $\{h_n\}_{n \in \mathbb{Z}} \subset C(X)$  be defined as

$$h_n(x) = \frac{1}{g_n(x)}, \quad x \in X.$$

Since  $g_n(x) \neq 0$  for every  $n \in \mathbb{Z}$  and for all  $x \in X$ , each function  $h_n$  is in  $C(X)$ , and hence the operators  $\rho(h_n)$  are well defined, bounded, linear operators.

Next, we notice that for  $n \in \mathbb{N}$ ,

$$\|h_n\| = \|1/g_n\| = \left\| \frac{\min_{0 \leq k \leq n} \tilde{p}_{0,k-1}(x)}{\tilde{p}_{0,n-1}(x)} \right\| \leq 1,$$

and that for  $n = -1, -2, \dots$ ,

$$\|h_n\| = \|1/g_n\| = \left\| \frac{\tilde{p}_{n,-1}(x)}{\min_{n \leq k \leq 0} \tilde{p}_{k,-1}(x)} \right\| = \left\| \frac{\tilde{p}_{n,-1}(x)}{\tilde{p}_{k,-1}(x)} \right\|$$

for some  $k$  satisfying  $n \leq k \leq 0$ , and therefore

$$\|h_n\| = \|\tilde{p}_{n,k-1}(x)\| \leq M.$$

Together with the fact that  $h_0(x) = 1$ , this shows that the operator

$$(5) \quad R = \text{diag}(\{\rho(h_n)\}_{n=-\infty}^{\infty},)$$

is a bounded linear operator. Since  $g_n h_n \equiv 1$  for all  $n$ ,  $SR = RS = I$ , which shows that  $S$  is an invertible operator whose inverse is  $R$ .

Finally, we show that  $S^{-1}TS$  is a contraction. Since  $S^{-1}TS$  is a bilateral (forward) operator weighted shift with weight sequence  $\{\rho(h_{n+1}f_n g_n)\}_{n \in \mathbb{Z}}$ , it suffices to prove that for all  $n \in \mathbb{Z}$  the operators  $\rho(h_{n+1}f_n g_n)$  are contractions. We notice that  $|f_n(x)| \leq \tilde{f}_n(x)$ , so it suffices to show that

$$\|h_{n+1}\tilde{f}_n g_n\| \leq 1, \quad n \in \mathbb{Z}.$$

Since

$$\|h_{n+1}\tilde{f}_n g_n\| = \sup_{x \in X} |h_{n+1}(x)\tilde{f}_n(x)g_n(x)| = \left| \frac{1}{g_{n+1}(x)} \tilde{f}_n(x)g_n(x) \right|,$$

it suffices to show that for any  $x$  in  $X$ ,

$$(6) \quad \frac{1}{g_{n+1}(x)} \tilde{f}_n(x)g_n(x) \leq 1.$$



Remark 2.1,  $\|S_n\| \leq \|\hat{S}_n\| \leq M$  and  $\|S_n^{-1}\| \leq \|\hat{S}_n^{-1}\| \leq M$ . Then it is easy to see that the operator  $S = \bigoplus_{n=1}^{\infty} S_n$  is a bounded invertible operator on  $\bigoplus_{n=1}^{\infty} \mathcal{H}_n$ , and that  $S$  satisfies  $\|S^{-1}TS\| \leq 1$  as well as  $\|S\|, \|S^{-1}\| \leq M$ .

Thus, for each of the types I, II, or III the similarity is bounded (uniformly) from above and from below, which shows that a direct sum of such operators is similar to a contraction. This completes the proof of Theorem 1.1. ■

#### 4. PROBLEMS

Since every centered operator is an arbitrary direct sum of operators of type I, II, III, and IV, it remains to investigate the case when a centered power bounded operator is of type IV. The structure of centered operators of type IV is set forth in Theorem 4.2 below, which is taken from [7].

If  $\mathcal{H}$  is a Hilbert space and if  $T \in \mathcal{L}(\mathcal{H})$  is a quasi-invertible operator with polar decomposition  $T = UP$ , then, let  $\theta_T$  denote the  $C^*$ -algebra generated by the operators  $\{(U^*)^n P U^n\}_{n=-\infty}^{+\infty}$  together with the identity operator. If  $X$  is a compact Hausdorff space, if  $\mu$  is a positive, regular, Borel measure on  $X$ , and if  $\mathcal{G}$  is a Hilbert space, then  $L_{\mathcal{G}}^2(\mu)$  will denote the Hilbert space of all measurable  $\mathcal{G}$ -valued functions on  $X$  which are square-integrable with respect to  $\mu$ . If  $\varphi \in L^\infty(\mu)$ , then  $M_\varphi$  will denote the multiplication operator on  $L_{\mathcal{G}}^2(\mu)$  defined by  $(M_\varphi f)(x) = \varphi(x)f(x)$  for all  $f \in L_{\mathcal{G}}^2(\mu)$ . An operator  $A$  on  $L_{\mathcal{G}}^2(\mu)$  is said to be *decomposable* if there is a bounded, measurable,  $\mathcal{L}(\mathcal{G})$ -valued function  $A(x)$  on  $X$  such that  $(Af)(x) = A(x)f(x)$  for every  $x \in X$  and every  $f \in L_{\mathcal{G}}^2(\mu)$ .

**Definition 4.1.** A type IV centered operator  $T$  will be called a *type IV<sub>n</sub> centered operator*,  $n = 1, 2, \dots, \infty$ , provided that  $\theta_T''$ , the abelian von Neumann algebra generated by  $\theta_T$ , has uniform multiplicity  $n$ .

Finally, we come to a model for type IV centered operators.

**Theorem 4.2.** *Let  $\mathcal{H}$  be a Hilbert space and let  $T \in \mathcal{L}(\mathcal{H})$  be a type IV centered operator. Then there exists a decomposition  $\mathcal{H} = \bigoplus_{i \in I} \mathcal{H}_i$  of  $\mathcal{H}$  consisting of reducing subspaces  $\mathcal{H}_i$  for  $T$  such that, relative to this decomposition,  $T|_{\mathcal{H}_i}$  is a centered operator of type IV<sub>n<sub>i</sub></sub>. Furthermore if  $T$  is a type IV<sub>n</sub> centered operator with polar decomposition  $T = UP$ ,  $X$  is the maximal ideal space of the commutative  $C^*$ -algebra  $\theta_T$ , and  $\Gamma$  denotes the Gelfand transform of  $\theta_T$  onto  $C(X)$ , then there exists*

- (1) a homeomorphism  $\tau$  of  $X$  onto  $X$ ,
- (2) a finite measure  $\mu$  on  $X$  which is quasi-invariant under  $\tau$ ,
- (3) an  $n$ -dimensional Hilbert space  $\mathcal{G}$ ,
- (4) a decomposable unitary operator  $\Theta$  on  $L_{\mathcal{G}}^2(\mu)$ , and
- (5) a Hilbert space isomorphism  $W$  from  $\mathcal{H}$  onto  $L_{\mathcal{G}}^2(\mu)$ ,

such that

$$WTW^{-1} = \Theta S M_{\Gamma(P)},$$

where  $S$  is the unitary operator defined on  $L_{\mathcal{G}}^2(\mu)$  by the formula

$$(Sf)(x) = f(\tau(x))(d(\mu \circ \tau)/d\mu)^{1/2}(x)$$

for all  $f \in L_{\mathcal{G}}^2(\mu)$ .

As it is easily seen, this model is much different from the models for centered operators of type I, II, and III, and it seems that it is necessary to apply somewhat different techniques in order to obtain an answer to the similarity problem.

As mentioned above it was shown in [10] that every polynomially bounded operator has a dilation that is weakly centered.

**Problem 4.3.** Is every weakly centered polynomially bounded operator similar to a contraction?

**Problem 4.4.** Does every polynomially bounded operator have a dilation that is centered?

Recall that an operator  $T$  in  $\mathcal{L}(\mathcal{H})$  is said to be  $n$ -normal if it is unitarily equivalent to an  $n \times n$  matrix with mutually commuting, normal entries. Obviously, each centered operator of type  $\text{III}_n$  is an  $n$ -normal operator.

**Problem 4.5.** Is every  $n$ -normal, polynomially bounded operator similar to a contraction?

*Note.* This paper is part of the author's Ph. D. thesis, written at the University of Michigan under the direction of Professor Carl Pearcy. The author gratefully acknowledges partial support by Texas A. & M. University during the period when this paper was being written.

*Remark 4.6.* Very recently the author, in collaboration with C. Pearcy and V. Paulsen, has proved that *every centered operator of type IV is similar to a contraction if and only if it is power bounded.* This result will appear in [9].

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