EQUILIBRIUM OF A PARTICLE

1. CONDITIONS FOR THE EQUILIBRIUM OF A PARTICLE

Newton's first law, which was introduced in Chapter I, states that a particle originally at rest, or moving in a straight line with constant velocity will remain in this state provided the particle is not subjected to an unbalance force. In other words, a particle will remain in *equilibrium* if the resultant of the forces acting on the particle is zero. Mathematically:

$$\sum \vec{F} = 0 \tag{3.1}$$

Since Newton's second law establishes that Eq. (3.1)

$$\sum \vec{F} = m\vec{a}$$

and from Eq. (3.1) the resultant of the forces acting on a particle is zero, it follows that

$$m\vec{a} = 0$$

that is, the acceleration of the particle is zero. Therefore Eq. (3.1) is a necessary and sufficient condition for the equilibrium of a particle.

3.2 THE FREE–BODY DIAGRAM

A *free–body diagram* will show all the forces acting on a particle. This is especially helpful when solving the equilibrium equation (Σ **F**). This diagram is simply a drawing of the particle isolated from its surroundings with *all* the forces that are acting *on* it.

Before establishing the steps necessary to sketch a free–body diagram it is important to briefly analyze two elements that are frequently used as connections, springs and pulleys.

Springs are usually employed to support loads or restraint the motion of bodies. In a large number of springs the displacement and the force applied to the element have a *linear* relation.

$$F \propto d \rightarrow F = k\,d$$

The above relation is known as *Hooke's law* and expresses that "*the magnitude of the force exerted on a linearly elastic spring is equal to the stiffness (constant) k of the spring times the distance d that the element is elongated (or compressed)*" (Textbook p. 84).

d is equal to $x_1 - x_o$, where x_o is the length of the spring before applying the load and x_1 is the length of the spring once the load has been applied.

Cables and Pulleys are elements used to transmit force or power and generally these elements are considered weightless. However, this simplification depends on the magnitude of force that is being transmitted with respect to the weight of the pulley or cable. It is also commonly assumed that cables and ropes cannot stretch. A cable can support only tension and this force acts always in the direction of the element.

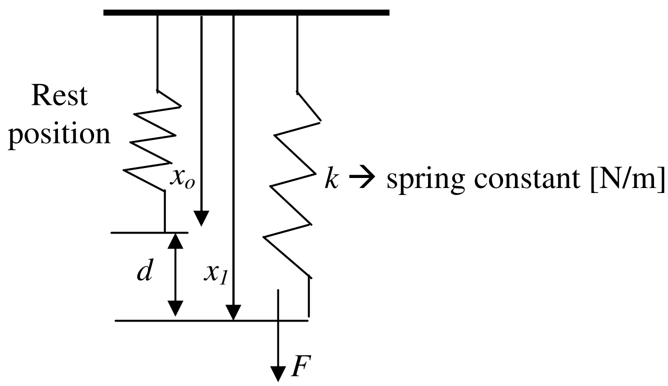

Figure 3.1. Force applied on a spring of constant k.

The tension developed in a cable which passes over a frictionless pulley must have a constant magnitude to maintain the equilibrium in the cable or rope. Figure 3.2 shows schematically this concept.

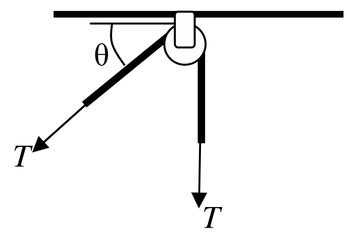

Figure 3.2. Cable supported by frictionless pulley subjected to tension T.

Figure 3.3(a) shows a system of two blocks connected by a rope that passes over a frictionless and weightless pulley. Figures 3.3(b, c, and d) present the FBD of blocks A, B and the pulley, respectively.

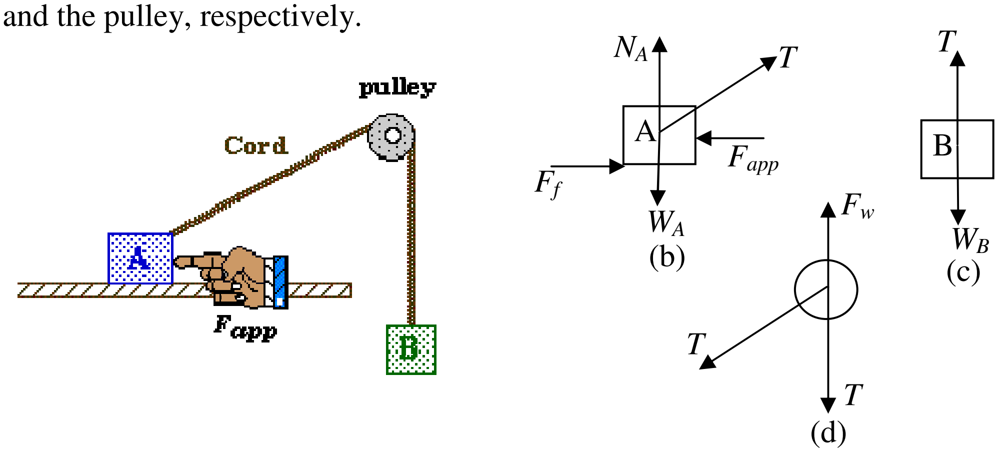

Figure 3.3. Example of free body diagram.

Notice that in the FBD's presented in figure 3.3, it has been assumed that the bodies involved in the system are particles which results in neglecting the geometry of the bodies of the system.

3.3 COPLANAR SYSTEM OF FORCES

When a particle is subjected to a system of coplanar forces, that is, forces that lie in the same plane, each force can be projected onto the *x* and *y* axes of the plane. Thus, if the forces acting on the particle are both coplanar and in equilibrium then Eq. (3.1)

$$\sum \vec{F} = 0$$

can be expressed as

$$\sum F_x \hat{\imath} + \sum F_y \hat{\jmath} = 0$$

Since both the *x* and *y* components of the previous equation are zero then

$$\sum F_x = 0 \qquad \sum F_y = 0 \tag{3.2}$$

Homework No. 3.1:
3–20, 3–23, 3–25, 3–27, 3–31.

3.4 THREE–DIMENSIONAL FORCE SYSTEM

The expression that must be satisfied to ensure the equilibrium for a particle was presented in Eq. (3.1)

$$\sum \vec{F} = 0 \tag{3.1}$$

Extending this expression to three dimensions results into

$$\sum F_x \hat{\imath} + \sum F_y \hat{\jmath} + \sum F_z \hat{k} = 0 \tag{3.3}$$

Thus, Eq. (3.3) can be decomposed into three equations, one for every direction as

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 \tag{3.4}$$

Homework No. 3.2:

3–48, 3–58, 3–60, 3–74.