STRUCTURAL ANALYSIS

7.1 SIMPLE TRUSSES

A truss is a structure composed of slender members joined together at their end points. The members commonly used in construction consist of wooden struts or metal bars. The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a *gusset plate*, as shown in figure 7.1(a).

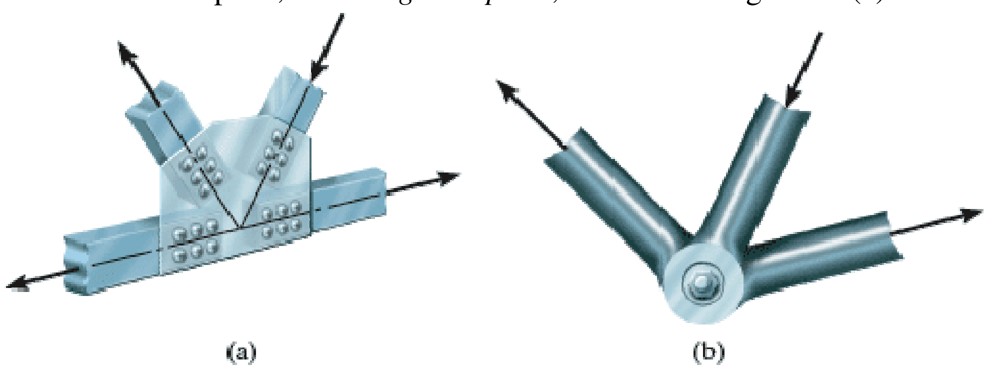

Figure7.1. Trusses commonly used in construction.

Planar Trusses

Planar trusses lie in a single plane and are often used to support roofs and bridges. The truss *ABCDE*, shown in figure 7.2(a), is an example of a typical roof-supporting truss. In this figure, the roof load is transmitted to the truss *at the joints* by means of a series of *purlins,* such as *DD'*. Since the imposed loading acts in the same plane as the truss, figure 7.2(b), the analysis of the forces developed in the truss members is two-dimensional.

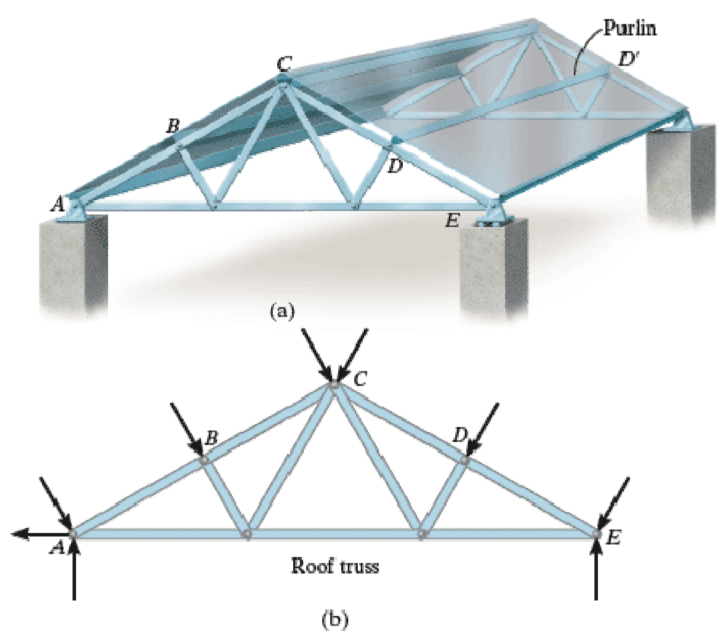

Figure 7.2. Planar trusses are commonly used to support bridges and roofs.

In the case of a bridge, figure 7.3(a), the load on the deck is first transmitted to *stringers,* then to *floor beams,* and finally to the *joints B, C*, and *D* of the two supporting side trusses. Like the roof truss, the bridge truss loading is also coplanar, figure 7.3(b).

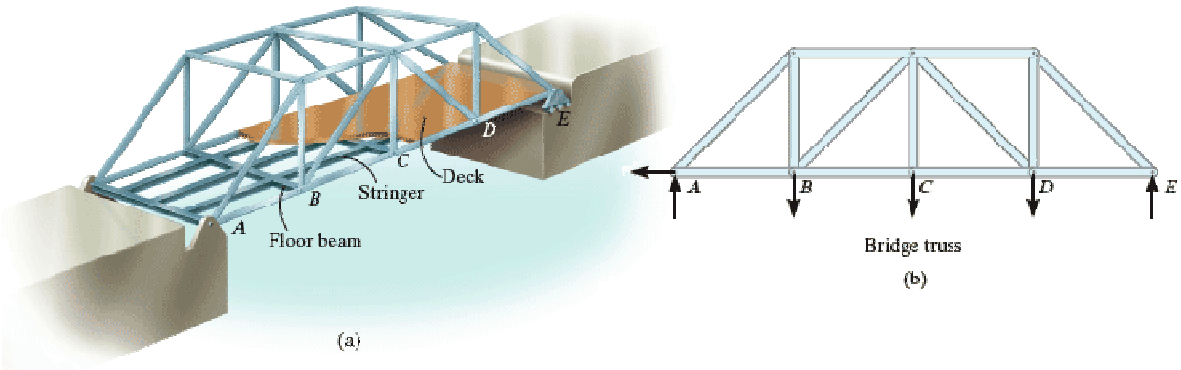

Figure 7.3. In several applications the forces involved in trusses are coplanar.

When bridge or roof trusses extend over large distances, a rocker or roller is commonly used for supporting one end. This type of support allows freedom for expansion or contraction of the members due to temperature or application of loads.

Assumptions for Design

To design both the members and the connections of a truss, it is first necessary to determine the *force* developed in each member when the truss is subjected to a given loading. In this regard, two important assumptions will be made:

1. *All loadings are applied at the joints.* In most situations, such as for bridge and roof trusses, this assumption is true. Frequently in the force analysis the weight of the members is neglected since the forces supported by the members are usually large in comparison with their weight. If the member's weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.

2. *The members are joined together by smooth pins.* In cases where bolted or welded joint connections are used, this assumption is satisfactory provided the center lines of the joining members are *concurrent,* as in Fig. 7.1(a).

Because of these two assumptions, *each truss member acts as a two-force member,* and therefore the forces at the ends of the member must be directed along the axis of the member. If the force tends to *elongate* the member, it is a *tensile force* (T), figure 7.4(a); whereas if it tends to *shorten* the member, it is a *compressive force* (C), figure 7.4(b). In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive. Often, compression members must be made *thicker* than tension members because of the buckling or column effect that occurs when a member is in compression.

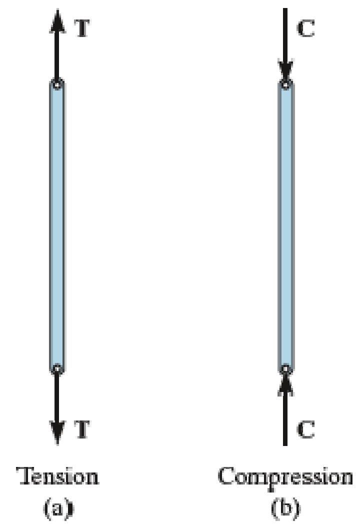

Figure 7.4. The members of a truss act as two – force members.

Simple Truss

To prevent collapse, the form of a truss must be rigid. Obviously, the four-bar shape *ABCD* in figure 7.5(a) will collapse unless a diagonal member, such as *AC*, is added for support. The simplest form that is rigid or stable is a *triangle*. Thus, a *simple truss* is constructed by *starting* with a basic triangular element, such as *ABC* in figure 7.5(b), and connecting two members (*AD* and *BD*) to form an additional element. As each additional element consisting of two members and a joint is placed on the truss, it is possible to construct a simple truss.

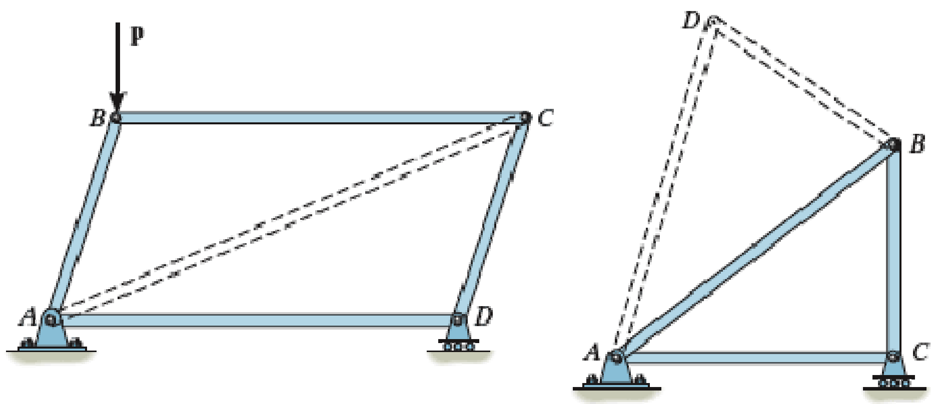

Figure 7.5. Construction of a simple truss.

7.2 THE METHOD OF JOINTS

The analysis or design of a truss requires the calculation of the force in each of its members. If a free–body diagram of the entire truss is sketched, the forces in the members are *internal forces;* therefore they cannot be obtained from an equilibrium analysis. Instead, by considering the equilibrium of a joint of the truss then a member force becomes an *external force* on the joint's free-body diagram, and the equations of equilibrium can be applied to obtain its magnitude. This forms the basis for the *method of joints*.

Because the truss members are all straight two-force members lying in the same plane, the force system acting at each joint is *coplanar and concurrent.* Consequently, rotational or moment equilibrium is automatically satisfied at the joint (or pin), and it is only necessary to satisfy $\Sigma F_x = 0$ and $\Sigma F_y = 0$ to ensure equilibrium.

When using the method of joints, it is *first* necessary to draw a free-body diagram of a joint before applying the equilibrium equations. To do this, recall that the *line of action* of each member force acting on the joint is *specified* from the geometry of the truss since the force in a member passes along the axis of the member. As an example, consider the pin at joint *B* of the truss in figure 7.6 (a). Three forces act on the pin, namely, the 500-N force and the forces exerted by members *BA* and *BC.* The free-body diagram is shown in figure 7.6 (b). As shown, $\mathbf{F}_{BA}$ is "pulling" on the pin, which means that member *BA* is in *tension;* whereas $\mathbf{F}_{BC}$ is "pushing" on the pin, and consequently member *BC* is in *compression.* These effects are clearly demonstrated by isolating the joint with small segments of the member connected to the pin, figure 7.6 (c). The pushing or pulling on these small segments indicates the effect of the member being either in compression or tension.

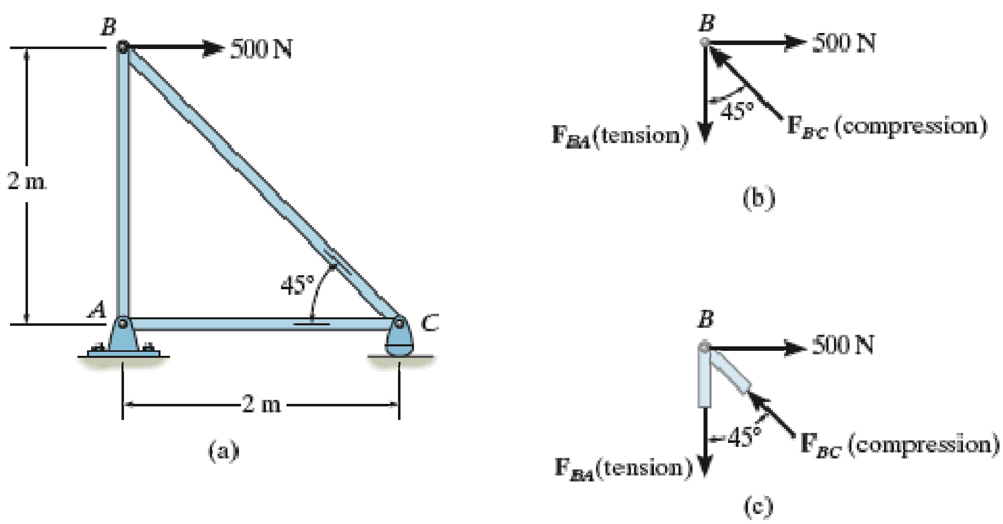

Figure 7.6. Process for analysis of a truss.

The analysis always should start at a joint having at least one known force and at most two unknown forces, as in figure 7.6 (b). In this way, application of $\Sigma F_x = 0$ and $\Sigma F_y = 0$ yields two algebraic equations which can be solved for the two unknowns. When applying these equations, the correct sense of an unknown member force can be determined using one of two possible methods:

1. *Always assume* the *unknown member forces* acting on the joint's free-body diagram to be in *tension,* i.e., "pulling" on the pin. If this is done, then numerical solution of the

equilibrium equations will yield *positive scalars for members in tension and negative scalars for members in compression.* Once an unknown member force is found, use its *correct* magnitude and sense (T or C) on subsequent joint free-body diagrams.

2. The *correct* sense of direction of an unknown member force can, in many cases, be determined "by inspection." For example, $\mathbf{F}_{BC}$ in figure 7.6 (b) must push on the pin (compression) since its horizontal component, F_{BC} sin 45°, must balance the 500-N force ($\Sigma Fx = 0$). Similarly, $\mathbf{F}_{BA}$ is a tensile force since it balances the vertical component in the *BC* element, F_{BC} cos 45° ($\Sigma Fy = 0$). In more complicated cases, the sense of an unknown member force can be *assumed;* then, after applying the equilibrium equations, the assumed sense can be verified from the numerical results. A *positive* answer indicates that the sense is *correct,* whereas a *negative* answer indicates that the sense shown on the free-body diagram must be *reversed.*

7.3 ZERO – FORCE MEMBERS

Truss analysis using the method of joints is greatly simplified if we can first determine those members which support *no loading*. These *zero-force members* are used to increase the stability of the truss during construction and to provide support if the applied loading is changed.

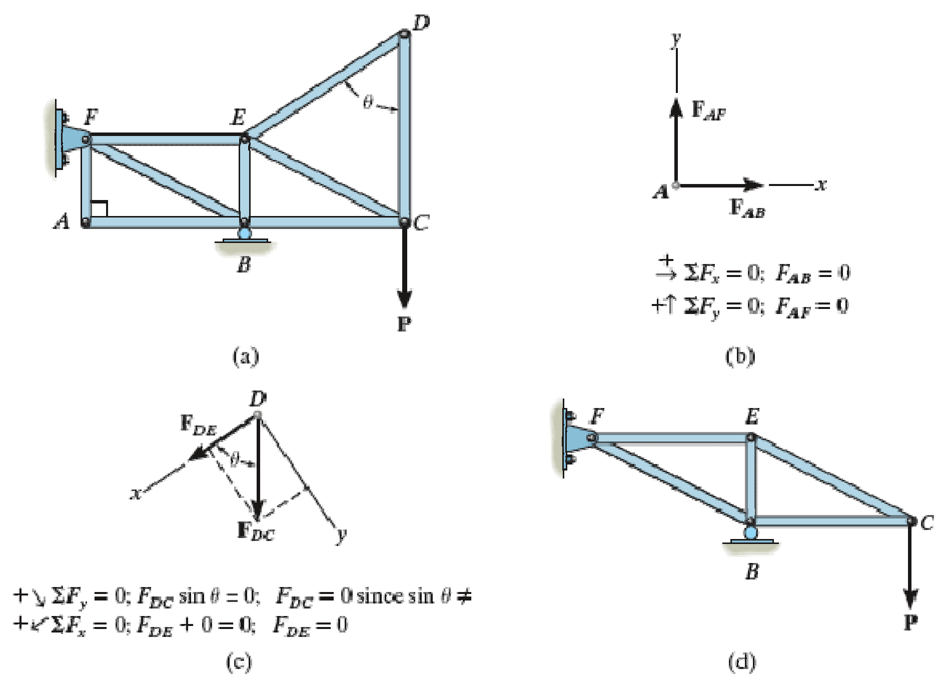

Figure 7.7. Procedure for determining zero – load elements.

The zero-force members of a truss can generally be determined *by inspection* of each of its joints. For example, consider the truss shown in figure 7.7 (a). If a free-body diagram of the pin at joint *A* is drawn, figure 7.7 (b), it is seen that members *AB* and *AF* are zero-force members. On the other hand, notice that we could not have come to this conclusion if we had considered the free-body diagrams of joints *F* or *B* simply because there are five unknowns at each of these joints. In a similar manner, consider the free-body diagram of joint *D*, figure 7.7 (c). Here again it is seen that *DC* and *DE* are zero-force members. As a general rule, *if only two members form a truss joint and no external load or support reaction is applied to the joint, the members must be zero-force members.* The load on the truss in figure 7.7 (a) is therefore supported by only five members as shown in figure 7.7 (d).

Now consider the truss shown in figure 7.8 (a). The free-body diagram of the pin at joint *D* is shown in figure 7.8 (b). By orienting the *y* axis along members *DC* and *DE* and the *x* axis along member *DA*, it is seen that *DA* is a zero-force member. Note that this is also the case for member *CA*, figure 7.8 (c). In general, *if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint*. The truss shown in figure 7.8 (d) is therefore suitable for supporting the load **P**.

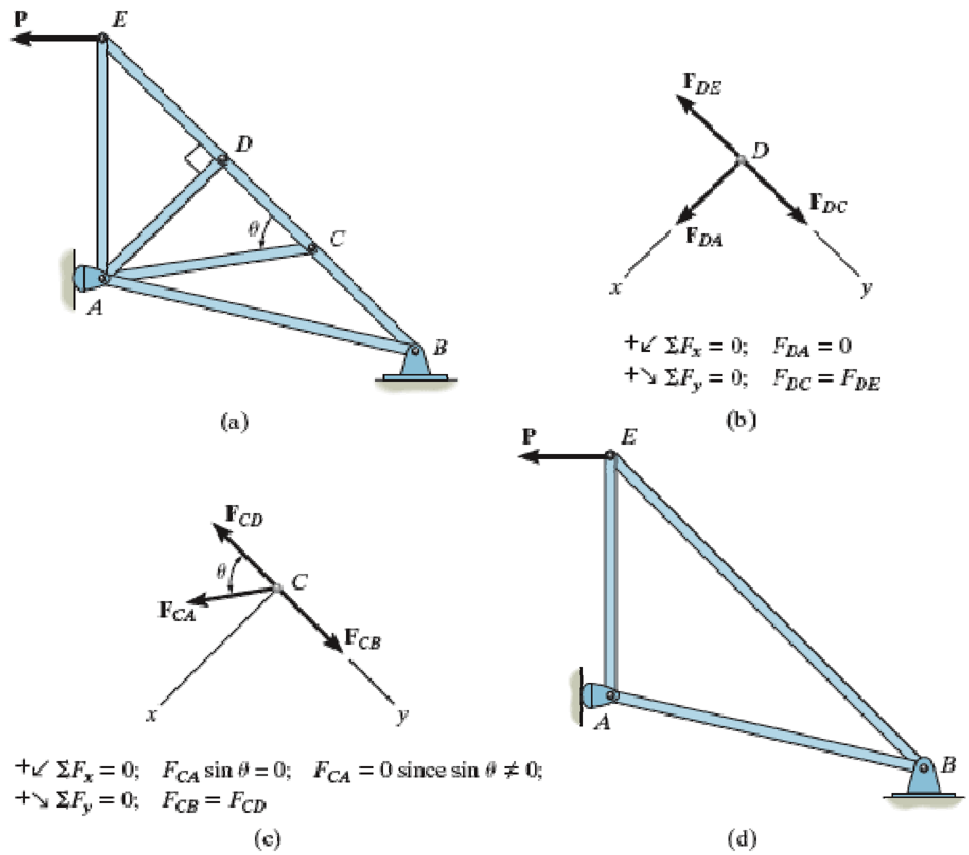

Figure 7.8. Procedure for determining zero – load elements in joints with three elements.

Homework No. 7.1:

6–7, 6–11, 6–26.

7.4 THE METHOD OF SECTIONS

When it is necessary to find the force in only a few members of a truss, it is possible to analyze the truss using the *method of sections*. It is based on the principle that if a body is in equilibrium then any part of the body is also in equilibrium. For example, consider the two truss members shown on the left in figure 7.9. If the forces within the members are to be determined, then an imaginary section indicated by the blue line, can be used to cut each member into two parts and thereby “expose” each internal force as “external” to the free-body diagrams shown on the right. Clearly, it can be seen that equilibrium requires that the member in tension (T) be subjected to a “pull,” whereas the member in compression (C) is subjected to a “push”.

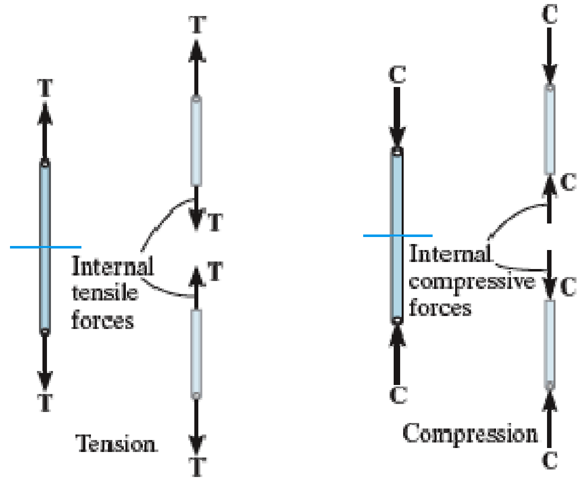

Figure 7.9. Principle of the method of sections for calculating the force exerted to specific members of a truss.

The method of sections can also be used to “cut” or section the members of an entire truss. If the section passes through the truss and the free-body diagram of either of its two parts is drawn, then the equations of equilibrium can be applied to that part to determine the member forces at the “cut section.” Since only *three* independent equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_O = 0$) can be applied to the isolated part of the truss, try to select a section that, in general, passes through not more than *three* members in which the forces are unknown. For example, consider the truss in figure 7.10 (a). If the force in member *GC* is to be determined, section *aa* would be appropriate. The free-body diagrams of the two parts are shown in figures 7.10 (b) and figure 7.10 (c). In particular, note that the line of action of each member force is specified from the *geometry* of the truss, since the force in a member passes along its axis. Also, the member forces acting on one part of the truss are equal but opposite to those acting on the other part—Newton’s third law. As noted above, the members assumed to be in *tension* (*BC* and *GC*) are subjected to a “pull”, whereas the member in *compression* (*GF*) is subjected to a “push”.

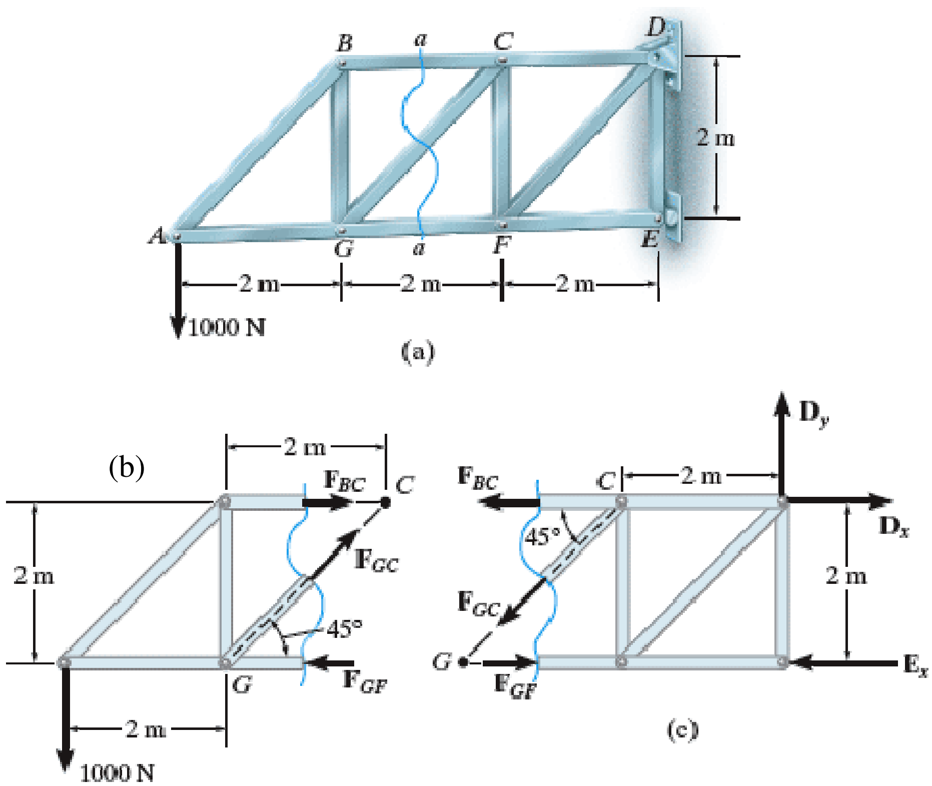

Figure 7.10. Procedure to apply the method of sections to a truss.

The three unknown member forces $\mathbf{F}_{BC}$, $\mathbf{F}_{GC}$, and $\mathbf{F}_{GF}$ can be obtained by applying the three equilibrium equations to the free-body diagram in figure 7.10 (b). If, however, the free-body diagram in figure 7.10 (c) is considered, the three support reactions $\mathbf{D}_x$, $\mathbf{D}_y$ and $\mathbf{E}_x$ will have to be known, because only three equations of equilibrium are available. (This, of course, is done in the usual manner by considering a free-body diagram of the *entire truss*).

When applying the equilibrium equations, one should consider ways of writing the equations so as to yield a *direct solution* for each of the unknowns, rather than having to solve simultaneous equations. For example, summing moments about *C* in figure 7.10 (b) would yield a direct solution for $\mathbf{F}_{GF}$ since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GC}$ create zero moment about *C*. Likewise, $\mathbf{F}_{BC}$ can be directly obtained by summing moments about *G*. Finally, $\mathbf{F}_{GC}$ can be found directly from a force summation in the vertical direction since $\mathbf{F}_{GF}$ and $\mathbf{F}_{BC}$ have no vertical components. This ability to *determine directly* the force in a particular truss member is one of the main advantages of using the method of sections.

Homework No. 7.2:

6–38, 6–41, 6–44, 6–49.

7.5 FRAMES

Frames and machines are two common types of structures which are often composed of pin-connected *multi-force members*, i.e., members that are subjected to more than two forces.

- *Frames* are generally stationary and are used to support loads.
- *Machines* contain moving parts and are designed to transmit and alter the effect of forces.

If a frame or machine is properly constrained and contains only the necessary supports or members to prevent collapse, the forces acting at the joints and supports can be determined by applying the equations of equilibrium to each member.

Free – Body Diagrams

In order to determine the forces acting at the joints and supports of a frame or machine, the structure must be *disassembled* and the free-body diagrams of its parts must be drawn. The following important points must be observed:

- Isolate each part by drawing its *outlined shape*. Then show all the forces and/or couple moments that act on the part.
- Identify all the two-force members in the structure and represent their free-body diagrams as having two equal but opposite collinear forces acting at their points of application. Adequate determination of two force members will avoid solving extra equations.
- Forces common to any two *contacting* members act with equal magnitudes but opposite sense on the respective members. If the two members are treated as a *"system" of connected members*, then these forces are *"internal"* and are *not shown* on the *free-body diagram of the system*; however, if the free-body diagram of *each member* is drawn, the forces are *"external"* and *must* be shown on each of the free-body diagrams.

Equations of Equilibrium

If the frame or machine is properly supported and contains only the supports or members that are necessary to prevent its collapse, then the unknown forces at the supports and connections can be determined from the equations of equilibrium. If the structure lies in the *x–y* plane, then for *each* free-body diagram drawn the loading must satisfy $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M_O = 0$.

Homework No. 7.3:

6–69, 6–74, 6–90, 6–103, 6–117, 6–119.