INTERNAL FORCES

8.1 INTERNAL FORCES DEVELOPED IN STRUCTURAL MEMBERS

Designing any mechanical member requires an investigation of the loads acting within the element to ensure that the material will be able to support such loads. The determination of these internal loads can be achieved by using the *method of sections*.

Considering a simple supported beam, as shown in figure 8.1*a*, subjected to two forces F_1 and F_2, and the support reactions A_x, A_y, and B_y (figure 8.1*b*). The determination of the *internal loadings* acting on the cross section at *C* can be achieved by considering that an imaginary section passes through the beam cutting it into two segments. By doing this, the internal loadings at the section *C* become *external* on the free–body diagram of each segment, as shown in figure 8.1*c*.

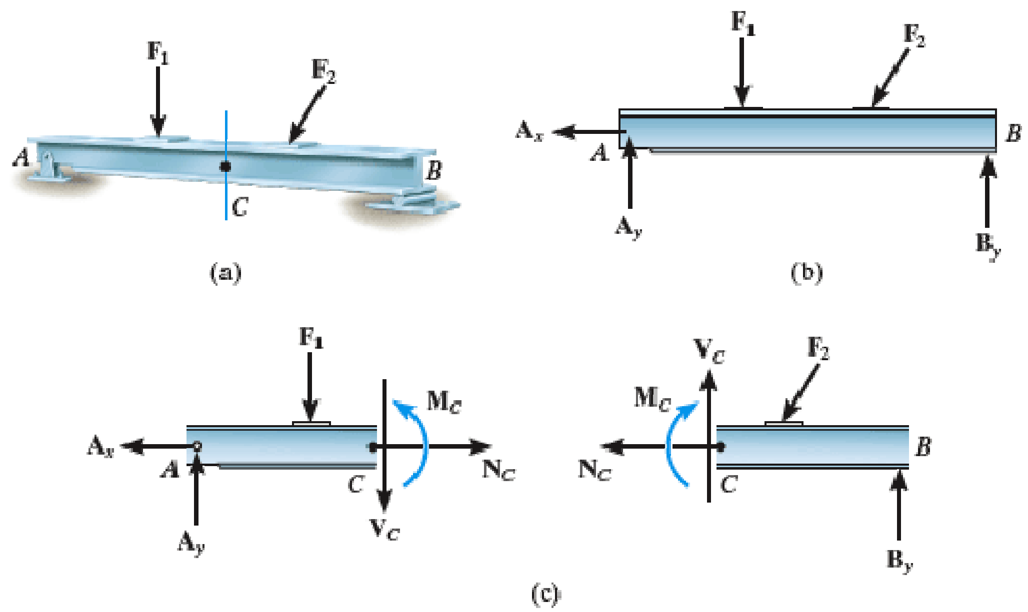

Figure 8.1. Determination of the internal forces in a beam.

Since segments *AC* and *CB* were in equilibrium *before* the beam was sectioned, the equilibrium of each segment is maintained provided the rectangular force components $\mathbf{N}_C$ and $\mathbf{V}_C$ and the resultant couple moment $\mathbf{M}_C$ are developed at the section. These loadings must be equal in magnitude and opposite in direction on each of the segments (Newton's third law). The magnitude of each of these loadings can be calculated by applying the three equations of equilibrium to either segment *AC* or *CB*. A *direct solution* for $\mathbf{N}_C$ is obtained by applying $\Sigma F_x = 0$; $\mathbf{V}_C$ is obtained directly from $\Sigma F_y = 0$; and $\mathbf{M}_C$ is determined by summing moments about point *C*, $\Sigma M_C = 0$, in order to eliminate the moments of the unknowns $\mathbf{N}_C$ and $\mathbf{V}_C$.

The force components **N**, acting normal to the beam at the cut section, and **V**, acting tangent to the section, are the *normal or axial force* and the *shear force*, respectively. The couple moment **M** is the *bending moment*, as presented in figure 8.2*a*. In three dimensions, a general internal force and couple moment resultant will act at the section. The *x, y, z* components of these loadings are shown in figure 8.2*b*. Here $\mathbf{N}_y$ is the *normal force*, and $\mathbf{V}_x$ and $\mathbf{V}_z$ are *shear force components*. $\mathbf{M}_y$ is a *torsional or twisting moment*, and $\mathbf{M}_x$ and $\mathbf{M}_z$ are *bending moment components*. For most applications, these resultant loadings will act at the geometric center or centroid (*C*) of the section's cross-sectional area. Although the magnitude for each loading generally will be different at various points along the axis of the member, the method of sections can always be used to determine their values.

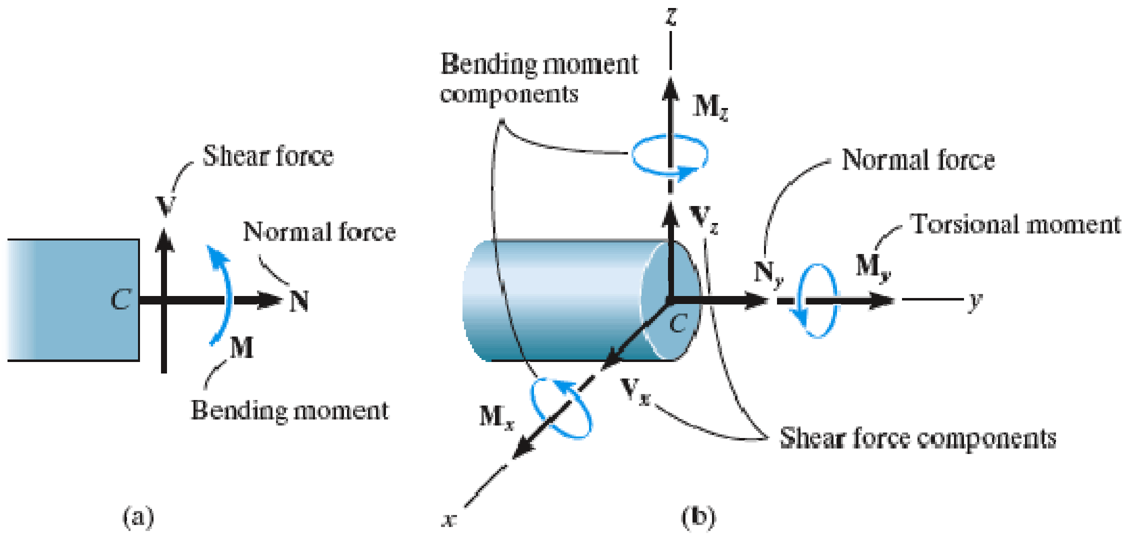

Figure 8.2. Normal and shear forces and torsional and bending moments.

FREE–BODY DIAGRAMS

Trusses are composed of two-force members that only support normal loads. On the other hand, frames and machines are composed of *multiforce* members, and so each of these members will generally be subjected to **internal normal, shear, and bending loadings**. For example, in order to determine the internal loadings in the frame shown in figure 8.3*a* at the sections cut by the line *H*, *G*, and *F*, the initial step would require to draw a free–body diagram of the top portion of this section as shown in figure 8.3*b*. At each point where a member is sectioned there is an unknown normal force, shear force, and bending moment, therefore, it is not possible to apply the three equations of equilibrium to this section in order to obtain these nine unknowns. Instead, to solve this problem it is necessary to disassemble the frame and determine the reactions at the connections of the members. Once this is done, each member may then be sectioned at its appropriate point, and the three equations of equilibrium can be applied to determine *N*, *V*, and *M*. For example, the free-body diagram of segment *DG*, figure 8.3c, can be used to determine the internal loadings at *G* provided the reactions of the pin, D_x and D_y are known.

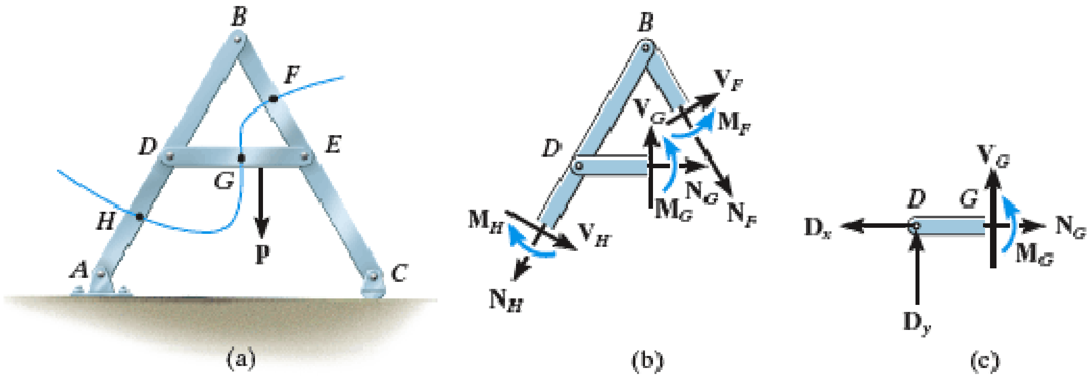

Figure 8.3. Determination of the internal forces in a structure.

Homework No. 8.1:

7–6, 7–11, 7–14, 7–24.

8.2 SHEAR AND MOMENT EQUATIONS AND DIAGRAMS

Beams are structural members designed to support loadings applied perpendicular to their axes. In general, beams are long, straight bars having a constant cross-sectional area. Often they are classified as to how they are supported. For example, a *simply supported beam* is pinned at one end and roller-supported at the other, as presented in figure 8.4*a*. On the other hand, a *cantilevered beam* is fixed at one end and free at the other. The actual design of a beam requires a detailed knowledge of the *variation* of the internal shear force V and bending moment M acting at *each point* along the axis of the beam. After this force and bending-moment analysis is complete, the theory of mechanics of materials and an appropriate engineering design code then can be used to determine the beam's required cross-sectional area.

The *variations* of V and M as functions of the position x along the beam's axis can be obtained by using the method of sections discussed in the previous section. The application of this method requires to section the beam at an arbitrary distance x from one end rather than at a specified point. If the results are plotted, the graphical variations of V and M as functions of x are termed the *shear diagram* and *bending-moment diagram*, respectively, this is presented in figures 8.4*b* and 8.4*c*.

The internal normal force will not be considered in the analysis performed here because in most of the cases the loads applied to a beam act perpendicular to the beam's axis and hence produce only an internal shear force and bending moment. An also, from the design viewpoint, the beam's resistance to shear, and particularly to bending, is more important than its ability to resist a normal force.

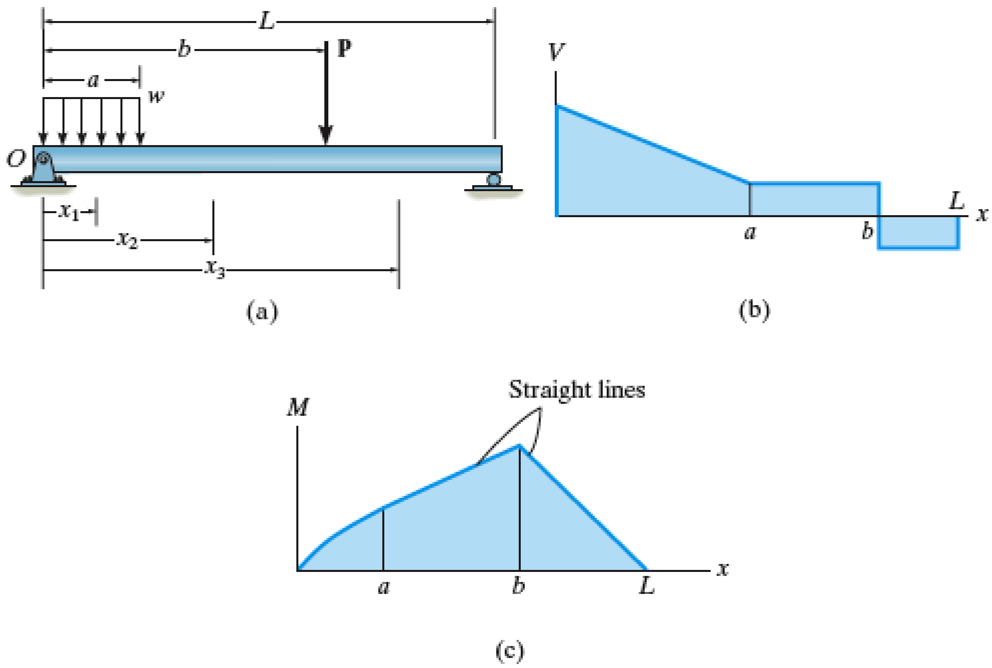

Figure 8.4. Shear force and bending moment diagrams.

Sign Convention

Although the choice of a sign convention is arbitrary, in the majority of engineering applications the convention explained next is followed. As schematically shown in figure 8.5, the positive directions are denoted by an internal *shear force* that causes *clockwise rotation* of the member on which it acts, and by an internal *moment* that causes *compression or pushing on the upper part* of the member. Also, positive moment would tend to bend the member if it were elastic, concave upward. Loadings that are opposite to these are considered negative

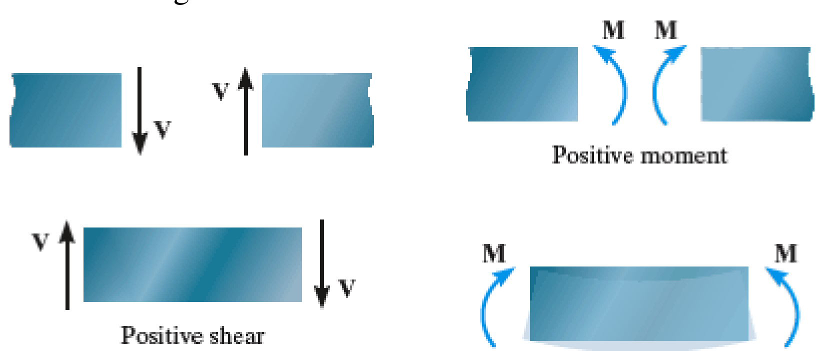

Figure 8.5. Beam sign convention.

Homework No. 8.2:

7–47, 7–50, 7–51, 7–56, 7–57.