
chapter

12

LINEARIZED SUPERSONIC FLOW

With the stabilizer setting at 2° the speed was allowed to increase to approximately 0.98 to 0.99 Mach number where elevator and rudder effectiveness were regained and the airplane seemed to smooth out to normal flying characteristics. This development lent added confidence and the airplane was allowed to continue until an indication of 1.02 on the cockpit Mach meter was obtained. At this indication the meter momentarily stopped and then jumped at 1.06, and this hesitation was assumed to be caused by the effect of shock waves on the static source. At this time the power units were cut and the airplane allowed to decelerate back to the subsonic flight condition.

Captain Charles Yeager, describing his flight on October 14, 1947—the first manned flight to exceed the speed of sound.

12.1 INTRODUCTION

The linearized perturbation velocity potential equation derived in Chapter 11, Equation (11.18), is

$$(1 - M_\infty^2) \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0 \quad [11.18]$$

and holds for both subsonic and supersonic flow. In Chapter 11, we treated the case of subsonic flow, where $1 - M_\infty^2 > 0$ in Equation (11.18). However, for supersonic flow, $1 - M_\infty^2 < 0$. This seemingly innocent change in sign on the first term of Equation (11.18) is, in reality, a very dramatic change. Mathematically, when $1 - M_\infty^2 > 0$ for subsonic flow, Equation (11.18) is an *elliptic* partial differential equation, whereas when $1 - M_\infty^2 < 0$ for supersonic flow, Equation (11.18) becomes a *hyperbolic*

differential equation. The details of this mathematical difference are beyond the scope of this book; however, the important point is that there *is* a difference. Moreover, this portends a fundamental difference in the physical aspects of subsonic and supersonic flow—something we have already demonstrated in previous chapters.

The purpose of this chapter is to obtain a solution of Equation (11.18) for supersonic flow and to apply this solution to the calculation of supersonic airfoil properties. Since our purpose is straightforward, and since this chapter is relatively short, there is no need for a chapter road map to provide guidance on the flow of our ideas.

12.2 DERIVATION OF THE LINEARIZED SUPERSONIC PRESSURE COEFFICIENT FORMULA

For the case of supersonic flow, let us write Equation (11.18) as

$$\lambda^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} - \frac{\partial \hat{\phi}}{\partial y^2} = 0 \quad [12.1]$$

where $\lambda = \sqrt{M_\infty^2 - 1}$. A solution to this equation is the functional relation

$$\hat{\phi} = f(x - \lambda y) \quad [12.2]$$

We can demonstrate this by substituting Equation (12.2) into Equation (12.1) as follows. The partial derivative of Equation (12.2) with respect to x can be written as

$$\frac{\partial \hat{\phi}}{\partial x} = f'(x - \lambda y) \frac{\partial(x - \lambda y)}{\partial x}$$

or
$$\frac{\partial \hat{\phi}}{\partial x} = f' \quad [12.3]$$

In Equation (12.3), the prime denotes differentiation of f with respect to its argument, $x - \lambda y$. Differentiating Equation (12.3) again with respect to x , we obtain

$$\frac{\partial^2 \hat{\phi}}{\partial x^2} = f'' \quad [12.4]$$

Similarly,

$$\frac{\partial \hat{\phi}}{\partial y} = f'(x - \lambda y) \frac{\partial(x - \lambda y)}{\partial y}$$

or
$$\frac{\partial \hat{\phi}}{\partial y} = f'(-\lambda) \quad [12.5]$$

Differentiating Equation (12.5) again with respect to y , we have

$$\frac{\partial^2 \hat{\phi}}{\partial y^2} = \lambda^2 f'' \quad [12.6]$$

