\[ q_\infty = \frac{1}{2} \rho_\infty V_\infty^2 \]

Dynamic pressure

Lift coefficient:
\[ C_L \equiv \frac{L}{q_\infty S} \]

Drag coefficient:
\[ C_D \equiv \frac{D}{q_\infty S} \]

Normal force coefficient:
\[ C_N \equiv \frac{N}{q_\infty S} \]

Axial force coefficient:
\[ C_A \equiv \frac{A}{q_\infty S} \]

Moment coefficient:
\[ C_M \equiv \frac{M}{q_\infty Sl} \]

Pressure coefficient:
\[ C_p = \frac{p - p_\infty}{q_\infty} \]

Skin friction coefficient
\[ C_f = \frac{\tau}{q_\infty} \]

The reference area \( S \) and reference length \( l \) are characteristic geometric quantities. The particular choice of them is not critical; however, when using force and moment coefficient data, you must always know what reference quantities the particular data are based upon.
Effects of Reynolds number on external flows

\[ \text{Re}_l = \frac{U l}{\nu} = \frac{\text{inertia effect}}{\text{viscosity effect}} \]

\[
\begin{cases} 
\text{Re} > 100 & \text{inertial effects dominate} \\
\text{Re} < 1 & \text{viscous effects dominate}
\end{cases}
\]

Small Re \(\#(0.1)\)

- Viscous effects very strong and felt far from the plate in all directions
- Inertial effect dominates the flow field. The viscous effects are restricted within a very thin layer (boundary layer) and the wake region
Not sufficiently streamlined objects when placed in flow field will tend to generate flow separation.
Boundary layers

\[ \text{Boundary layer thickness } \delta = \gamma \quad \text{where} \quad u = 0.99 \ U \]

\[ \text{Boundary layer displacement thickness: } \delta^* = \int_{0}^{\infty} \left( 1 - \frac{u}{U} \right) \, dy \]

**B.L. displacement thickness**

Represents the outward displacement of the streamlines caused by the Viscous effects on the plate.
\[ \delta^* b u = \int_0^\infty (u - u_b) db. \]

\[ \int_0^\infty (u - u_b) db = \int_0^\infty u_b db - \int_0^\infty u db \]

U doesn't change with y since it's the freestream velocity far from the boundary layer.

RHS = \( u_b y - \int_0^\infty u_b db = \delta^* b u \)

\[ \Rightarrow \int_0^\infty u db = u_b y - \delta^* b u = u_b(y - \delta^*) \]

If 2-D, \( b \) = per unit

\[ \int_0^\infty u db = u_b(y - \delta^*) \]

---

the flowrate across section is the same as that for a uniform flow with velocity \( U \) through a duct whose walls have been moved inward by \( \delta^* \).
Air flowing into a 2-ft-square duct with a uniform velocity of 10 ft/s forms a boundary layer on the walls as shown in Fig. E9.3. The fluid within the core region (outside the boundary layers) flows as if it were inviscid. From advanced calculations it is determined that for this flow the boundary layer displacement thickness is given by

$$\delta^* = 0.0070(x)^{1/2}$$

where $\delta^*$ and $x$ are in feet. Determine the velocity $U = U(x)$ of the air within the duct but outside of the boundary layer.

The B.L. displacement thickness is defined in terms of volumetric flow rate

$$U_1A_1 = 10 \text{ ft/s} \times (2 \text{ ft})^2 = 40 \frac{\text{ft}^3}{\text{s}} = \int u \, dA$$

$$40 \frac{\text{ft}^3}{\text{s}} = \int (2 \text{ ft} - 2\delta^*) \, dA = U(2 \text{ ft} - 2\delta^*)^2$$

$$U = \frac{10}{(1 - 0.0070x^{1/2})^2} \text{ ft/s}$$

The velocity increases in the downstream direction. Why?

The viscous effects reduce the effective size of the duct, causing the fluid to accelerate.
Boundary layer momentum thickness

If we concern momentum flux across section \( b-b \) rather than the volumetric flow rate, we will obtain B. L. momentum thickness

\[
\int \rho u (U - u) \, dA = \rho b \int_0^\infty u (U - u) \, dy
\]

\[
\rho b U^2 \Theta = \rho b \int_0^\infty u (U - u) \, dy
\]

\[
\Theta = \int_0^\infty \frac{u}{U} \left( 1 - \frac{u}{U} \right) \, dy
\]

Boundary layer thickness \( \delta = y \) where \( u = 0.99 \ U \)

Boundary layer displacement thickness \( \delta^+ = \int_0^\infty \left( 1 - \frac{u}{U} \right) \, dy \)

Boundary layer momentum thickness \( \Theta = \int_0^\infty \frac{u}{U} \left( 1 - \frac{u}{U} \right) \, dy \)
Blasius boundary layer solution for a flat plate flow

\[
\begin{align*}
\frac{\delta}{x} & = \frac{5}{\sqrt{\text{Re}_x}} \\
\frac{\delta^{*}}{x} & = \frac{1.721}{\sqrt{\text{Re}_x}} \\
\Theta & = \frac{0.664}{\text{Re}_x} \\
\tau_w & = 0.332 U^{3/2} \sqrt{\frac{p\mu}{x}}
\end{align*}
\]

Boundary layer thickness proportional to the square root of \(x\) location. \(\delta \sim \sqrt{x}\)

Drag acting on a flat plate

\[
\sum F_x = \rho \int_1 uV \cdot \mathbf{n} \, dA + \rho \int_2 uV \cdot \mathbf{n} \, dA
\]

\[
\sum F_x = -\mathcal{D} = -\int_{\text{plate}} \tau_w \, dA = -b \int_{\text{plate}} \tau_w \, dx
\]

\[
-\mathcal{D} = \rho \int_1 U(-U) \, dA + \rho \int_2 u^2 \, dA
\]

\[
\mathcal{D} = \rho U^2 bh - \rho b \int_0^\delta u^2 \, dy
\]

Mass conservation

\[
\rho U^2 bh = \rho b \int_0^\delta U u \, dy
\]

\[
\mathcal{D} = \rho b \int_0^\delta u(U - u) \, dy
\]

\[
\tau_w = \rho U^2 \frac{d\Theta}{dx}
\]

\[
\mathcal{D} = \rho b U^2 \Theta
\]
Consider the laminar flow of an incompressible fluid past a flat plate at $y = 0$. The boundary layer velocity profile is approximated as $u = Uy/\delta$ for $0 \leq y \leq \delta$ and $u = U$ for $y > \delta$, as is shown in Fig. E9.4. Determine the shear stress by using the momentum integral equation. Compare these results with the Blasius results given by Eq. 9.18.

\[ \tau_w = \rho U^2 \frac{d\Theta}{dx} \]

\[ \Theta = \int_0^\infty \frac{u}{U} \left( 1 - \frac{u}{U} \right) dy = \int_0^\delta \frac{u}{U} \left( 1 - \frac{u}{U} \right) dy = \int_0^\delta \left( \frac{y}{\delta} \right) \left( 1 - \frac{y}{\delta} \right) dy = \frac{\delta}{6} \]

At the same time $\tau_w = \mu \frac{U}{\delta}$ hence, \[ \frac{\mu U}{\delta} = \rho U^2 \frac{d\delta}{dx} \implies \delta = 3.46 \sqrt{\frac{\nu x}{U}} \implies \tau_w = 0.289 U^{3/2} \sqrt{\frac{\rho \mu}{x}} \]

<table>
<thead>
<tr>
<th>Profile Character</th>
<th>$\delta Re_x^{1/2}/x$</th>
<th>$c_f Re_x^{1/2}$</th>
<th>$C_D Re_x^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Blasius solution</td>
<td><strong>5.00</strong></td>
<td>0.664</td>
<td>1.328</td>
</tr>
<tr>
<td>b. Linear $u/U = y/\delta$</td>
<td>3.46</td>
<td>0.578</td>
<td>1.156</td>
</tr>
<tr>
<td>c. Parabolic $u/U = 2y/\delta - (y/\delta)^2$</td>
<td>5.48</td>
<td>0.730</td>
<td>1.460</td>
</tr>
<tr>
<td>d. Cubic $u/U = 3(y/\delta)/2 - (y/\delta)^3/2$</td>
<td>4.64</td>
<td>0.646</td>
<td>1.292</td>
</tr>
<tr>
<td>e. Sine wave $u/U = \sin[\pi(y/\delta)/2]$</td>
<td>4.79</td>
<td>0.655</td>
<td>1.310</td>
</tr>
</tbody>
</table>
Turbulent flows

The value of $Re$ at transition from laminar flow and for the flow to become turbulent are complex functions of many parameters.

Turbulent flow problems are
- everywhere in ME and AE,
- solved by experiments and computer simulations.

No analytical solution available.

Turbulent velocity files are more fuller, more like the ideal uniform profile.
Effect of pressure gradient

Decrease (increase) in pressure in the flow direction is termed a favorable (adverse) pressure gradient.

If no viscosity, the flow will be symmetric and no drag will be generated.

In experiments, however the drag are always observed no matter how small the viscosity is.

The drag must be due to viscosity

Important: external flow could be thought of as consisting two layers, i.e. viscous B. L. and inviscid freestream. Pressure distribution is imposed on B. L. by the outer freestream. Pressure within the B. L. is that given by the inviscid flow field. Pressure is constant in the direction normal to the flow direction within B. L.

In real flows over a blunt body, because of the viscous dissipation, fluid particles don’t have sufficient energy as those in inviscid flows to climb up the pressure hill (C to F). The separation of flow will happen.
Effect of pressure gradient

For flow separation, the average pressure on the rear half of the cylinder is considerably less than that on the front half. Large pressure drag is developed, even though the viscous shear drag may be quite small.

When the angle of attack becomes large, the flow tends to separate from the top surface of the airfoil, creating a large wake of relatively “dead air” behind the airfoil. Under such conditions, the lift will decreases and the drag will increases dramatically. It is said the airfoil is stalled. Very dangerous!

Figure 4.4 Schematic of lift-coefficient variation with angle of attack for an airfoil.
Drag and lift

Most of the information is a result of numerous experiments with wind tunnel, water tunnels etc.
Drag and lift

A model wing of constant chord length is placed in a low-speed subsonic wind tunnel, spanning the test section. The wing has an NACA 2412 airfoil and a chord length of 1.3 m. The flow in the test section is at a velocity of 50 m/s at standard sea-level conditions. If the wing is at a 4° angle of attack, calculate (a) $c_l$, $c_d$, and $c_m,c/4$ and (b) the lift, drag, and moments about the quarter chord, per unit span.

Solution

a. From App. D, for an NACA 2412 airfoil at a 4° angle of attack,

\[
\begin{align*}
c_l &= 0.63 \\
c_m,c/4 &= -0.035
\end{align*}
\]

To obtain $c_d$, we must first check the value of the Reynolds number:

\[
Re = \frac{\rho \infty V \infty c}{\mu \infty} = \frac{(1.225 \text{ kg/m}^3)(50 \text{ m/s})(1.3 \text{ m})}{1.789 \times 10^{-5} \text{ kg/(m)(s)}} = 4.45 \times 10^6
\]

For this value of Re and for $c_l = 0.63$, from App. D,

\[
c_d = 0.007
\]

b. Since the chord is 1.3 m and we want the aerodynamic forces and moments per unit span (a unit length along the wing, perpendicular to the flow), $S = c(1) = 1.3(1) = 1.3 \text{ m}^2$. Also

\[
q_\infty = \frac{1}{2} \rho \infty V^2_\infty = \frac{1}{2}(1.225)(50)^2 = 1531 \text{ N/m}^2
\]

From Eq. (5.22),

\[
L = q_\infty S c_l = 1531(1.3)(0.63) = 1254 \text{ N}
\]

Since 1 N = 0.2248 lb, also

\[
L = (1254 \text{ N})(0.2248 \text{ lb/N}) = 281.9 \text{ lb}
\]

\[
D = q_\infty S c_d = 1531(1.3)(0.007) = 13.9 \text{ N}
\]

\[
= 13.9(0.2248) = 3.13 \text{ lb}
\]

Note: The ratio of lift to drag, which is an important aerodynamic quantity, is

\[
\frac{L}{D} = \frac{c_l}{c_d} = \frac{1254}{13.9} = 90.2
\]

\[
M_{c/4} = q_\infty S c_m,c/4 = 1531(1.3)(-0.035)(1.3)
\]

\[
M_{c/4} = -90.6 \text{ N-m}
\]
In 1977 the *Gossamer Condor* won the Kremer prize by being the first human-powered aircraft to complete a prescribed figure-of-eight course around two turning points 0.5 mi apart (Ref. 22). The following data pertain to this aircraft:

- flight speed $= U = 15$ ft/s
- wing size $= b = 96$ ft, $c = 7.5$ ft (average)
- weight (including pilot) $= W = 210$ lb
- drag coefficient $= C_D = 0.046$ (based on planform area)
- power train efficiency $= \eta = \text{power to overcome drag/pilot power} = 0.8$

Determine the lift coefficient, $C_L$, and the power, $\mathcal{P}$, required by the pilot.

For steady flight conditions the lift must be exactly balanced by the weight, or

$$W = L = \frac{1}{2} \rho U^2 AC_L$$

Thus,

$$C_L = \frac{2W}{\rho U^2 A}$$

where $A = bc = 96 \times 7.5 = 720$ ft$^2$, $W = 210$ lb, and $\rho = 2.38 \times 10^{-3}$ slugs/ft$^3$ for standard air. This gives

$$C_L = \frac{2(210 \text{ lb})}{(2.38 \times 10^{-3} \text{ slugs/ft}^3)(15 \text{ ft/s})^2(720 \text{ ft}^2)} = 1.09$$

A reasonable number. The overall-lift-to-drag ratio for the aircraft is $C_L/C_D = 1.09/0.046 = 23.7$. The product of the power that the pilot supplies and the power train efficiency equals the useful power needed to overcome the drag, $\mathcal{D}$. That is,

$$\eta \mathcal{P} = \mathcal{D} U$$

where

$$\mathcal{D} = \frac{1}{2} \rho U^2 AC_D$$

Thus,

$$\mathcal{P} = \frac{\mathcal{D} U}{\eta} = \frac{\frac{1}{2} \rho U^2 AC_D U}{\eta} = \frac{\rho AC_D U^3}{2\eta}$$

$$\mathcal{P} = \frac{(2.38 \times 10^{-3} \text{ slugs/ft}^3)(720 \text{ ft}^2)(0.046)(15 \text{ ft/s})^3}{2(0.8)}$$

$$\mathcal{P} = 166 \text{ ft} \cdot \text{lb/s} \left(\frac{1 \text{ hp}}{550 \text{ ft} \cdot \text{lb/s}}\right) = 0.302 \text{ hp}$$

(Ans)