

GRAPHS & DIGRAPHS

5th Edition

Gary Chartrand

Western Michigan University

Linda Lesniak

Drew University

Ping Zhang

Western Michigan University

Preface to the fifth edition

Since graph theory was considered to have begun some 275 years ago, it has evolved into a subject with a fascinating history, a host of interesting problems and numerous diverse applications. While graph theory has developed ever-increasing connections with other areas of mathematics and a variety of scholarly fields, it is its beauty that has attracted so many to it.

As with the previous editions, the objective of this fifth edition is to describe much of the story that is graph theory – in terms of its concepts, its theorems, its applications and its history. Here too, the audience for the fifth edition is beginning graduate students and advanced undergraduate students. The main prerequisite required of students using this book is a knowledge of mathematical proofs. Some elementary knowledge of linear algebra and group theory is also useful for some topics.

Although a one-semester course in graph theory using this text can be designed by selecting topics of greatest interest to the instructor and students, there is more than ample material available for a two-semester sequence in graph theory. Our goal has been to prepare a book that is interesting, carefully written, student-friendly and consisting of clear proofs. The fifth edition is approximately 50% longer than the fourth edition. Some major changes from the fourth edition are:

- (1) sections have been divided into subsections to make the material easier to read and locate;
- (2) terms being defined are in bold type, making them easier to locate;
- (3) more than 300 new exercises have been added;
- (4) examples and applications have been added to illustrate concepts and theorems;

(5) historical discussions of mathematicians and problems have been expanded.

There is a section at the end of the book giving hints and solutions to odd-numbered exercises, providing information on one possible approach that may be useful to solve the problem. There is expanded or new coverage of a number of topics, including

- degree sequences
- toughness
- graph minors
- perfect graphs
- chromatic polynomials
- list colorings and list edge colorings
- nowhere-zero flows
- flows in networks.

Over the years, there have been some changes in notation that a number of mathematicians now use. When certain notation appears to have been adopted by sufficiently many mathematicians working in graph theory so that this has become the norm, we have adhered to these changes. In particular,

- a path is now expressed as $P = (v_1, v_2, \dots, v_k)$ and a cycle as $C = (v_1, v_2, \dots, v_k, v_1)$;
- the Cartesian product of two graphs G and H is expressed as $G \square H$, rather than the previous $G \times H$;
- the union of G and H is expressed by $G + H$, rather than $G \cup H$;
- the join of two graphs G and H is expressed as $G \vee H$, rather than $G + H$.

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G.C., L.L. and P.Z.

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