

Chapter 6

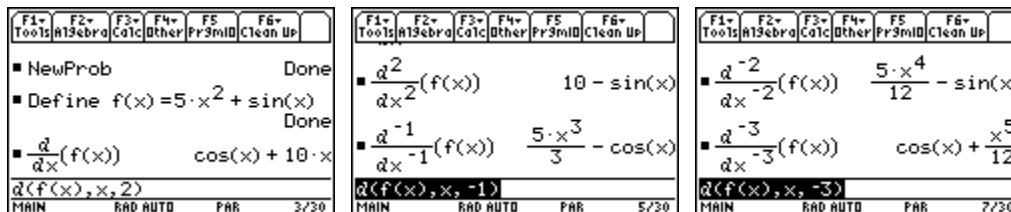
Antiderivatives

- §1. Two Notations for Antiderivatives
- §2. Riemann Sums and the Sigma Notation
- §3. Symbolic Integrals

For the activities in this chapter, the various graphing, numerical, and symbolic capabilities of the calculator will be used to explore antiderivatives. This is loosely coordinated with Chapter 6 of the text *Calculus with Early Vectors*, by Phillip Zenor, Edward Slaminka, and Donald Thaxton, Prentice Hall, 1999.

1. Two Notations for Antiderivatives

Interestingly, there are two different ways that we can get the TI-89 to give a symbolic antiderivative. The first way we present here relates naturally to the command for finding higher order derivatives. Recall $d(f(x),x,2)$ is the command for getting the second derivative. The last argument in this command is the order of the derivative desired. (If you leave off this last argument, you get a first derivative by default.) Simply use order -1 to get the antiderivative. Other negative integers give repeated antidifferentiation. Notice the interesting (and logical) notation for an antiderivative in the history area. Also note that we get only one antiderivative (and not the family of all possible antiderivatives).



The textbook notation for an antiderivative is actually the most common. It relates to the relationship we will soon learn between definite integrals and antiderivatives. This command notation (called integration) is also available. The commands to differentiate and integrate appear on the keyboard (as 2nd functions above 8 and 7) and in the F3 Calculus menu.



Using the integration command with only the first two arguments gives an antiderivative (not the family of all possible antiderivatives). Giving an undefined variable name (as we did above with "c") will give an antiderivative with this undefined variable name added to the result. This provides a way for us to represent the whole family of all antiderivatives. Later in the chapter,

you will learn the significance of further arguments in the integration command ([LO,UP] in the catalog help line above).

2. Riemann Sums and the Sigma Notation

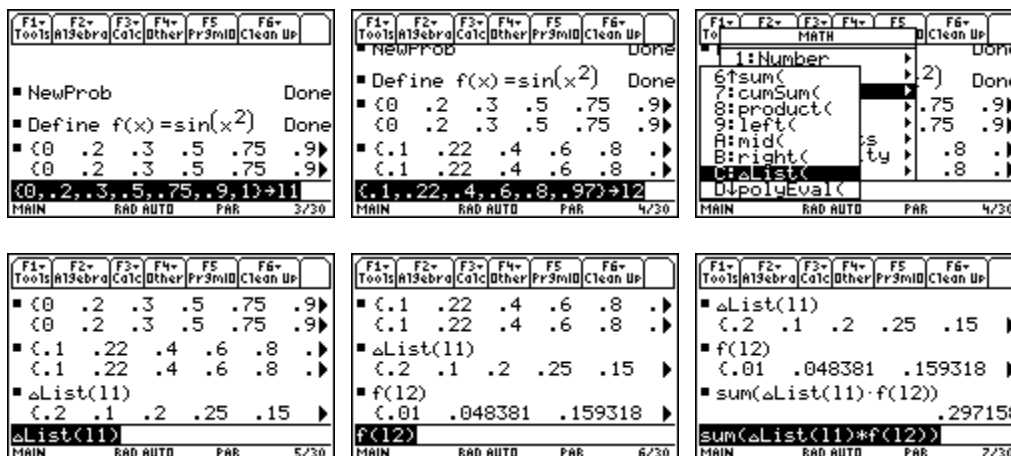
Definite integrals are defined by means of a limit of Riemann sums, and one of the ways to approximate a definite integral is to use some kind of Riemann sum. There are several ways to compute Riemann sums on the TI-89/Voyage 200 family of calculators.

A general Riemann sum approximating $\int_a^b f(x)dx$ is defined in terms of a partition $\{x_i\}$ of the interval $[a,b]$ into subintervals $[x_0, x_1], [x_1, x_2], \dots, [x_{i-1}, x_i], \dots, [x_{n-1}, x_n]$ and in terms of a selection of evaluation points $\{s_i\}$ with $s_i \in [x_{i-1}, x_i]$ for $i=1, 2, \dots, n$. Namely,

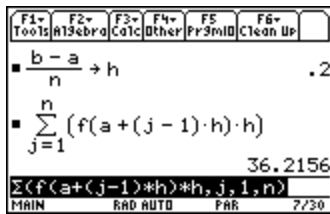
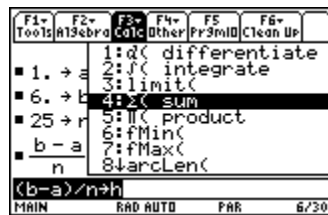
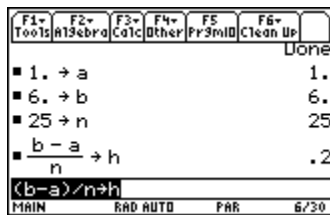
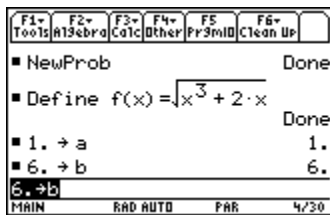
$$\int_a^b f(x)dx \approx \sum_{i=1}^n f(s_i)(x_i - x_{i-1}).$$

Each term in the sum can be interpreted as the area of a rectangle with width $(x_i - x_{i-1})$ and “signed” height $f(s_i)$. Special cases include selecting $s_i = x_{i-1}$ (left endpoints), $s_i = x_i$ (right endpoints), or $s_i = \frac{x_{i-1} + x_i}{2}$ (midpoints). Often the partition points $\{x_i\}$ are equally spaced across the interval, giving $(x_i - x_{i-1}) = \Delta x$ for all $i=1, 2, \dots, n$. First we show how to compute a general Riemann sum (no special assumptions) using lists.

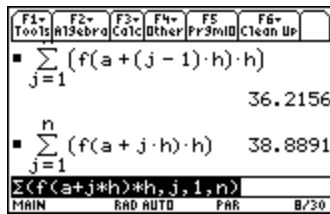
Start by storing the formula for $f(x)$. Store the partition points $\{x_i\}$ in a list, say I1. (Note that variable names must begin with a letter, and so the first character in the name of this list is the letter “I”, not the number “1”.) Store the selection points $\{s_i\}$ in another list, say I2. Then take advantage of the list operations to compute the Riemann sum in the HOME screen. Applying a function to a list gives a list of evaluations. The command Δ List computes a list of the differences (i.e. $(x_i - x_{i-1})$). Multiplying two lists of equal length, gives a list where like terms in the two lists have been multiplied. Finally the sum command sums the terms in a list.



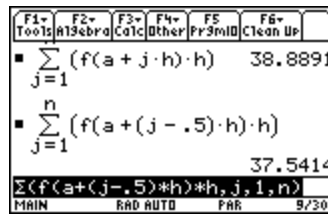
If you desire an equally spaced partition with special evaluation points (left endpoints, right endpoints, midpoints), there are simpler ways to have the calculator compute these.



Left endpoints

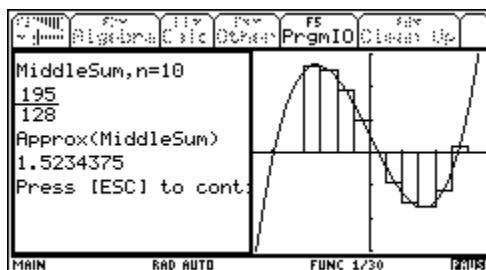
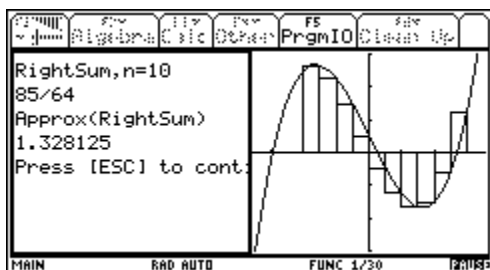
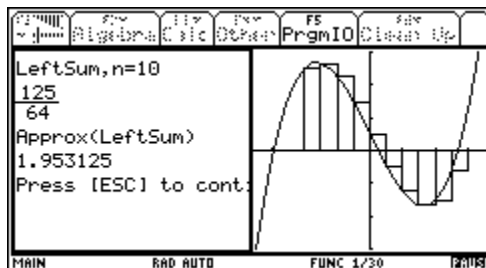
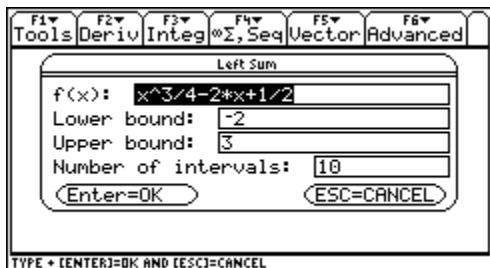
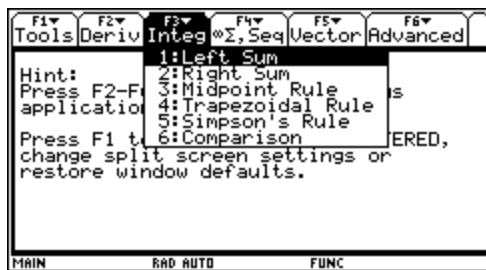
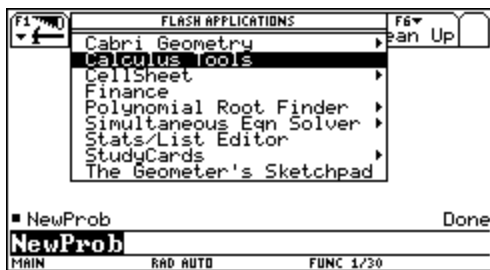


Right endpoints



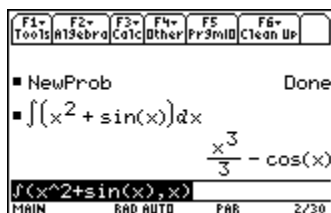
Midpoints

The free Flash App called Calculus Tools has nice implementations of these Riemann sums as well.

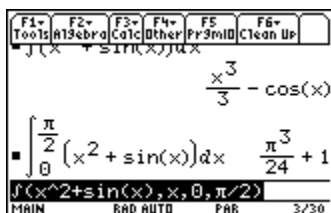


3. Symbolic Integrals

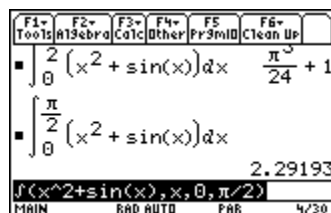
The Fundamental Theorem of Calculus shows how definite integrals and antiderivatives are related. Thus another name used for an antiderivative is *indefinite integral*. We have already seen this integration command used to find antiderivatives in the first section of this chapter. The same command will find definite integrals if we specify the interval of integration. Make sure the Exact/Approximate mode is set to AUTO or EXACT so that we can see exact integration answers.



Indefinite Integral

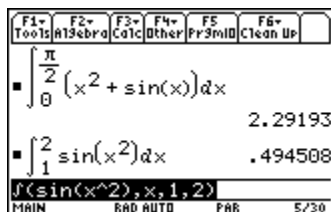


Exact Definite Integral
[Press ENTER.]

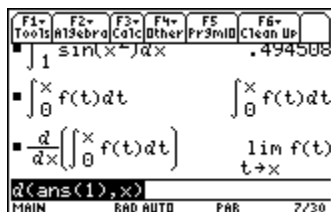


Approximate Definite Integral
[Press \blacklozenge ENTER.]

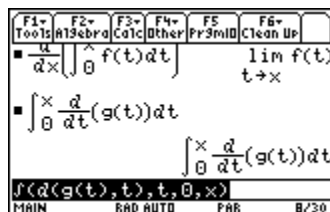
Thus you can use the calculator to check your hand work using the Fundamental Theorem of Calculus. Usually the calculator can do a symbolic problem if you can do it by hand. If the integrand in a definite integral does not have an elementary antiderivative (or the calculator cannot find one), the AUTO mode will automatically switch to a numerical approximation (using a method a little more accurate and efficient than Riemann sums).



[Press ENTER.]



Using unknown f



Using unknown g