

If you do not have this flash application, then it is easy to write short programs to do these computations.

The text (by Zenor, Slaminka, and Thaxton) uses the idea of the trapezoidal rule and Simpson's rule in a slightly more complicated way to actually approximate antiderivatives. This corresponds to one step of the trapezoidal rule (Theorem 1) and one step of Simpson's rule (Theorem 4). Perhaps it will be easiest if we think of a situation where the integrand $f(x)$ is known only by a table of discrete values. This is often the case for scientific data. If you happen to have a formula for the function $f(x)$, then you can easily create a discrete list of this form.

For example, consider $f(x) = \sin(x^2)$ for $1 \leq x \leq 2$. The choice of $n = 10$ we made above corresponds to $h = \frac{b-a}{n} = \frac{2-1}{10} = 0.1$. The command `seq(1+j*0.1,j,0,10)` will generate a list of the desired x -values on this interval $[1, 2]$. Taking f of this list will give the desired evaluations.



View in Stats/List Editor (free Flash App)

This information will most often be displayed in a book or on your paper in a table.

x	$f(x)$
1.0	0.841471
1.1	0.935616
1.2	0.991458
1.3	0.992904
1.4	0.925212
1.5	0.778073
1.6	0.549355
1.7	0.248947
1.8	-0.098249
1.9	-0.451466
2.0	-0.756802

We are interested in approximating $\int_1^x f(t) dt$ for the same set of x -values as used in the table above, using only the numbers available in the table. This is easy to do for $x = 1$ because

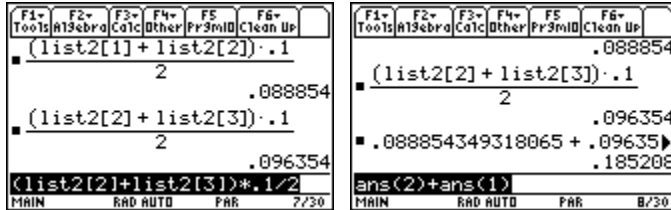
$$\int_1^1 f(t) dt = 0. \text{ For } x = 1.1, \text{ we use one trapezoid to approximate } \int_1^{1.1} f(t) dt \approx \frac{f(1) + f(1.1)}{2} (0.1)$$

and get 0.088854. In a similar fashion, we can use another trapezoid to approximate

$$\int_{1.1}^{1.2} f(t) dt \approx \frac{f(1.1) + f(1.2)}{2} (0.1) \approx 0.096354. \text{ Thus to get}$$

$$\int_1^{1.2} f(t) dt \approx 0.088854 + 0.096354 \approx 0.185208.$$

All of this can be done in the home screen as illustrated below. If the number n is small, these step-by-step calculations are reasonable to do.



Obviously what we want to do is to automate this process a little. The command

$$\text{seq}((\text{list2}[j]+\text{list2}[j+1])*0.1/2,j,1,10)$$

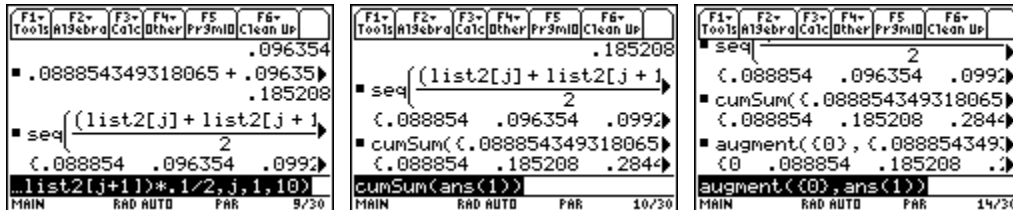
will generate a sequence of numbers representing the individual trapezoid approximations

$$\int_{x_j}^{x_{j+1}} f(t) dt \approx \frac{f(x_j) + f(x_{j+1})}{2} (0.1).$$

The MATH menu, List submenu command $\text{cumSum}(\text{above list})$ will give the cumulative sum we desire to approximate

$$\int_1^{x_{j+1}} f(t) dt \approx \sum_{k=1}^j \frac{f(x_k) + f(x_{k+1})}{2} (0.1).$$

Finally we append the initial zero to the list to get a final list of the same length as the original lists. This command is $\text{augment}(\{0\}, \text{above list})$



x	$f(x)$	$\int_1^x f(t) dt$ (Trapezoidal)
1.0	0.841471	0
1.1	0.935616	0.088854
1.2	0.991458	0.185208
1.3	0.992904	0.284426
1.4	0.925212	0.380332
1.5	0.778073	0.465496
1.6	0.549355	0.531868
1.7	0.248947	0.571783
1.8	-0.098249	0.579318
1.9	-0.451466	0.551832
2.0	-0.756802	0.491418

Notice that the final answer in the last column is the Trapezoidal Rule computation from the Calculus Tools choice we did at the very beginning.

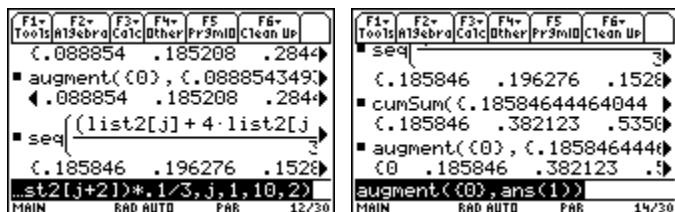
Implementing the textbook Simpson rule step-by-step is similar. The one different feature here is that our new column is not complete. The approximation we use is

$$\int_{x_j}^{x_{j+2}} f(t) dt \approx \frac{f(x_j) + 4f(x_{j+1}) + f(x_{j+2})}{3}(h),$$

giving us only partial definite integrals over “double” subintervals.

```
augment(cumSum(seq((list2[j]+4*list2[j+1]+list2[j+2])*0.1/2,j,1,10,2)))
```

Notice that we use an additional argument in the sequence command to indicate that the index j goes from 1 to 10 in steps of 2.



x	$f(x)$	$\int_1^x f(t) dt$ (Simpson)
1.0	0.841471	0
1.1	0.935616	
1.2	0.991458	0.185846
1.3	0.992904	
1.4	0.925212	0.382123
1.5	0.778073	
1.6	0.549355	0.535018
1.7	0.248947	
1.8	-0.098249	0.583248
1.9	-0.451466	
2.0	-0.756802	0.494551

Again notice that the value at the bottom of the last column is the Simpson rule approximation from Calculus Tools that we did at the very beginning of this section.

You may want to put these steps into a short program or save the history as a text file to more quickly do the step-by-step trapezoidal rule and the step-by-step Simpson’s rule repeatedly for new problems.

The discrete computations in this section work very nicely in a spreadsheet. If you have the CellSheet (it comes on the Voyage 200), you can experiment with alternative ways to

accomplish the step-by-step trapezoidal rule and step-by-step Simpson's rule in the spreadsheet that is available on this device.

2. Other Numerical Integration

There are more efficient and accurate algorithms for numerical integration than the composite trapezoidal rule and the composite Simpson's rule. One feature often put in these more sophisticated algorithms is an adaptive process to estimate the error made for the particular problem given. The algorithm then adjusts the step-size h as it works along the interval to achieve a desired accuracy. TI uses such an adaptive numerical integration routine internally, and it works to try to achieve 6 significant digits of accuracy on this family of calculators. Thus when display digits is set to the default FLOAT 6 mode, you can generally expect all of the displayed digits in a numerical integration computation to be correct. If you display more digits than 6, you cannot assume all the additional digits are correct.

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$f(x) \quad \sin(x^2)$ $nInt(f(x), x, 1, 2) \quad .494508$ $\int_1^2 f(x) dx \quad .494508$ $nInt(f(x), x, 1, 2)$					
MAIN RAD AUTO PAR 2/30					

It is generally better to use the internal numerical integration (either the $nInt$ (command or the regular integration command with a \blacklozenge ENTER) rather than the composite trapezoidal rule or the composite Simpson's rule. The best advice is to use one of these simple numerical integration routines only when the specific routine is specified. Generally in a textbook problem or a test question, the instruction will tell you not only the method (trapezoidal or Simpson) but also the step size.

3. Logarithmic and Exponential Functions

The TI-89/Voyage 200 offers the natural logarithm function and the natural exponential function on the keyboard. Many people do not know that the common logarithm function (base 10) is also available in the CATALOG as \log . Notice what happens when you try to do derivatives using a common logarithm or an exponential function with base 10.

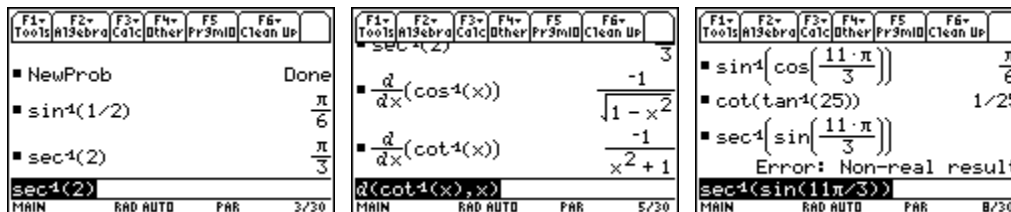
F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{d}{dx}(\ln(x)) \quad \frac{1}{x}$ $\frac{d}{dx}(\ln(x))$					
Base e					

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\frac{d}{dx}(e^x) \quad e^x$ $\frac{d}{dx}(e^x)$					
Base e					

F1 Tools	F2 Algebra	F3 Calc	F4 Other	F5 Pr3mID	F6 Clean Up
$\log(8.92) \quad .950365$ $\frac{d}{dx}(\log(x)) \quad \frac{1}{\ln(10) \cdot x}$ $\frac{d}{dx}(10^x) \quad \ln(10) \cdot 10^x$ $\frac{d}{dx}(10^x)$					
Base 10					

4. Inverse Trigonometric Functions

If you have upgraded your OS to at least 2.08 or higher, then you have not only all of the trigonometric functions, but also all of the inverse trigonometric functions as well. The ones not printed on the keyboard can be found in the CATALOG.

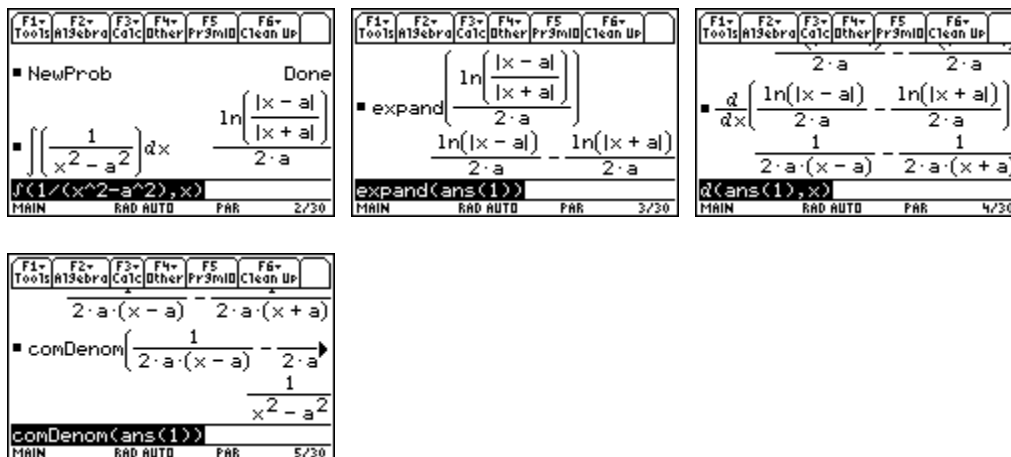


Notice that you will get the error message “Non-real result” if you get outside of the domain of some of the inverse trigonometric functions (assuming your MODE setting for Complex Format is REAL). All of the trigonometric functions have generalizations to domains of complex numbers, and the calculator is programmed to handle complex numbers when the Complex Format in the MODE screen is selected to something other than REAL.

5. Comparing Symbolic Integration to Tables of Integrals

Our text presents a short table of integral (and many others do as well). Before computer algebra systems were available, books were printed with very extensive tables of known indefinite and definite integrals. Now it is more common to rely on the expertise of a computer algebra system (or CAS for short). The TI-89/Voyage 200 OS can do essentially all of the integrals that can be handled by using the table in the text. It is useful to compare how the results might look a little different.

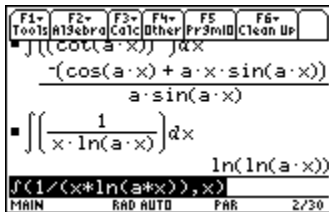
The text comments on page 377 about different systems can give different looking results for $\int \frac{dx}{x^2 - a^2}$ (Number 95, page 384). Here we try it on the TI-89. Notice we can always differentiate the result to check.



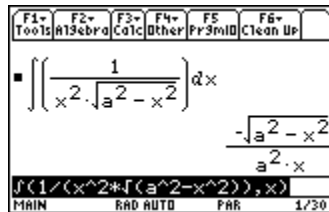
Here we try a few more.



Number 15, page 379



Number 46, page 381



Number 82, page 383

Some of the formulas in the table do not give a final answer, but rather reduce the task to another in the table or a simpler integral of the same type. For example Number 18, page 379 concerning an n -th power of the sine reduces the task to the $(n - 2)$ -nd power. Used repeatedly, we eventually can complete the task using Number 6 or 12. The best that we can do to “test out” the formula on the calculator is to try specific integers, such as $n = 4$ and 5. For simplicity, we also use $a = 1$. Because the results are two long to fit in the screen without scrolling, we reproduce them here nicely typeset.

$$\int ((\sin(x))^4) dx = \left(\frac{-(\sin(x))^3}{4} - \frac{3 \cdot \sin(x)}{8} \right) \cdot \cos(x) + \frac{3 \cdot x}{8}$$

$$\int ((\sin(x))^5) dx = \left(\frac{-(\sin(x))^4}{5} - \frac{4 \cdot (\sin(x))^2}{15} - 8/15 \right) \cdot \cos(x)$$