Dynamic concentration in random greedy processes

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Abstract

Independent sets and matchings in hypergraphs are objects of fundamental interest in combinatorics. Many classical problems can be framed as problems about these objects. Thus it is natural to consider algorithms that produce large matchings or large independent sets.

Let $H$ be a $D$-regular, $r$-uniform hypergraph on $n$ vertices, where $r$ is fixed and $D$ tends to infinity as $n$ goes to infinity. In this series of talks we consider the random greedy algorithms for producing matchings and independent sets on $H$. The random greedy matching algorithm builds a matching by adding edges selected uniformly at random one at a time, subject to the condition that each selected edge does not intersect any previously selected edge. The random greedy independent set algorithm builds an independent set by adding vertices one at a time which are chosen uniformly at random subject to the condition that the collection of selected vertices never contains an edge of $H$.

We will survey some recent results on these algorithms, explore some of the psuedorandom properties of the objects these algorithms produce, review the implications these results have for some classical questions in combinatorics, and discuss some striking questions that remain open.