Semantic limits of combinatorial objects

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The theory of limits of dense discrete combinatorial objects has gained considerable attention in the last years, not only by proving itself a valuable tool in the study of extremal combinatorics problems, but also for its own interesting problems. The syntactic, algebraic approach to the subject is popularly known as "flag algebras", while the semantic, geometric one is often associated with the name "graph limits". While the latter approach yields a more intuitive and expressible language, as its name suggests, it only applies to the theory of graphs. For other combinatorial objects such as digraphs, hypergraphs, permutations or posets, there have been several ad hoc semantic limit constructions in the literature.

In this talk, I will present a semantic limit for dense combinatorial objects in the same general setting as flag algebras. I will briefly state the main results associated with this semantic construction and I will illustrate how semantics can make syntactic arguments more natural.

This talk is based on joint work with Alexander Razborov.

Recent problems on saturation of hypergraphs

Sean English
Ryerson University

For a graph F, we say a hypergraph H is Berge-F if it can be obtained from F be replacing each edge of F with a hyperedge containing it. We say a hypergraph is Berge-F-saturated if it does not contain a Berge-F, but adding any hyperedge creates a copy of Berge-F. The k-uniform saturation number of Berge-F, $\mathrm{sat}_k(n, \text{Berge-}F)$ is the fewest number of edges in a Berge-F-saturated k-uniform hypergraph on n vertices. The saturation problem can be viewed as a minimization version of the traditional Turán problem, in

which one attempts to maximize the number of edges in a F-saturated graph.

In this talk we will explore recent problems involving the saturation numbers for Berge hypergraphs. We will cover specific numbers for nice families of graphs, as well as general bounds on the growth rates of these numbers. We will also discuss many open problems in this area.

Schubert polynomials and the strong Sperner property

Zachary Hamaker
University of Michigan

Recently, Gaetz and Gao proved the strong Sperner property for the weak order of the symmetric group using an SL_2 action. We give an alternative characterization of their lowering operator using derivatives of Schubert polynomials. In addition to simplifying Gaetz and Gao's proof, this allows us to prove a Stanley's conjectural formula for the determinant of a certain matrix and also leads to a simple proof of an identity of Macdonald's.

The work presented is joint with Oliver Pechenik, David Speyer and Anna Weigandt.

Introduction to flag algebras and applications

Bernard Lidický
Iowa State University

Flag algebras is a recent tool developed by Razborov for solving problems in extremal graph theory and combinatorics. The tool is very general and it lead to breakthrough results on many long standing open problems. It was applied in the area of graphs, hypergraphs, permutations, Ramsey numbers, discrete geometry and phylogenetic trees to name a few areas. The method provides results in the limit, which can be translated to large structures with small error terms. The method is also closely related to the notion of dense graph limits. In the series of three talks we first develop some intuition for the machinery by introducing it as random sampling in large graphs. Then we do a formal approach and examples. We describe a series of applications and techniques that are often used in conjunction with flag algebras. We also discuss which problems have a good potential for the method and which are not a good fit.

Polynomial to exponential transition in Ramsey theory

Dhruv Mubayi
University of Illinois at Chicago

After a brief introduction to classical hypergraph Ramsey numbers, I will focus on the following problem. What is the minimum t such that there exist arbitrarily large k-uniform hypergraphs whose independence number is at most polylogarithmic in the number of vertices and every s vertices span at most t edges? Erdős and Hajnal conjectured (1972) that this minimum can be calculated precisely using a recursive formula and Erdős offered $500 for a proof. For $k = 3$, this has been settled for many values of s, but it was not known for larger k. Here we settle the conjecture for all k at least 4. Our method also answers a question of Bhatt and Rödl about the maximum upper density of quasirandom hypergraphs.

This is joint work with Alexander Razborov.

An isoperimetric inequality for the Hamming cube and some consequences

Jinyoung Park
Rutgers University

I will introduce an isoperimetric inequality for the Hamming cube and some of its applications. The applications include a “stability” version of Harper’s edge-isoperimetric inequality, which was first proved by Friedgut, Kalai and Naor for half cubes, and later by Ellis for subsets of any size. Our inequality also plays a key role in a recent result on the asymptotic number of maximal independent sets in the cube.

This is joint work with Jeff Kahn.

Extremal problems for hypergraph expansions

Jacques Verstraete
University of California San Diego

If F is an r-uniform hypergraph, then the Turán number $\mathrm{ex}(n, F)$ denotes the maximum number of edges in an n-vertex r-uniform hypergraph that does not contain F, and the Ramsey number $r(n, F)$ denotes the minimum N such that every N-vertex r-uniform hypergraph contains either an independent set of size n or a copy of F. In this talk I will survey recent results on Turán-type and Ramsey-type questions for "sparse" uniform hypergraphs, which include trees and cycles. We discuss a variety of methods, employing techniques from diverse areas of mathematics. A notable theorem for hypergraph "expansions" includes as special cases classical results such as the Erdős-Ko-Rado Theorem.
