1. Evaluate the integral \( \int_{0}^{1} xe^{x} \, dx \).

2. Evaluate the integral \( \int_{0}^{\pi/2} \sin^{3} x \, dx \).

3. Evaluate the integral \( \int_{1}^{\sqrt{3}} \frac{\sqrt{x^{2} + 1}}{x^{4}} \, dx \).

4. Evaluate the integral \( \int \frac{dx}{x^{3} + x^{2} + x + 1} \).

5. Use Comparison Theorem to determine whether the integral \( \int_{1}^{\infty} \frac{dx}{x^{3} + 1} \) is convergent or divergent.

6. The base of a certain solid is the region enclosed by \( y = 1/x \), \( y = 0 \), \( x = 1 \), and \( x = 4 \). Every cross section of the solid taken perpendicular to the \( x \)-axis is an isosceles right triangle with its hypotenuse across the base. Find the volume of the solid.

7. If a force of 20 pounds is required to hold a spring 1 ft beyond its unstressed length, how much work does it take to stretch the spring this far?

8. Find the area of the region enclosed by the curves \( y = \sin x \), \( y = \cos x \), and \( x = 0 \).

9. Find the length of the curve \( x = y^{3/2}/3 - y^{1/2} \) from \( y = 1 \) to \( y = 9 \).

10. Find the average value of \( f(x) = \sin x \) on the interval \( [0, \pi/2] \).
Solutions

1. Integration by parts: \( u = x, \ dv = e^x \, dx \). Result: 1.

2. Substitution \( u = \cos x \). Then \( \sin x \, dx = -du \) and \( \sin^2 x = 1 - u^2 \). Obtain
\[
\int_1^0 (1 - u^2)(-du). \text{ Result: } 2/3.
\]

3. Substitution \( x = \tan t \). Then \( \sin x \, dx = -du \) and \( \sin^2 x = 1 - u^2 \). Obtain
\[
\int_{\pi/4}^{\pi/3} \frac{\sqrt{\sec^2 t}}{\tan^4 t} \sec^2 t \, dt = \cdots = \int_{\pi/4}^{\pi/3} \frac{\cos t}{\sin^4 t} \, dt. \text{ Now substitution } u = \sin t \text{ leads to } \int_{\sqrt{2/3}}^{\sqrt{3/2}} \frac{du}{u^4}.
\]
Result: \( \frac{8}{6\sqrt{2}} - \frac{8}{9\sqrt{3}} \).

4. Factor \( x^3 + x^2 + x + 1 = (x + 1)(x^2 + 1) \). Use partial fractions decomposition
\[
\frac{1}{(x + 1)(x^2 + 1)} = \frac{1/2}{x + 1} - \frac{-x/2 + 1/2}{x^2 + 1}. \text{ Result: } \frac{1}{2} \ln |x + 1| - \frac{1}{4}(x^2 + 1) + \frac{1}{2} \tan^{-1} x + C.
\]

5. Use the inequality \( \frac{1}{x^3 + 1} < \frac{1}{x^3} \) and the fact that \( \int_1^\infty \frac{dx}{x^3} \) converges as a \( p \)-integral with \( p = 3 \). Conclusion: the given integral converges.

6. The hypothenuse is \( 1/x \) so each leg is \( 1/(x\sqrt{2}) \). The area of the cross section is
\[
A(x) = 1/(4x^2). \text{ The volume is } \int_1^4 \frac{dx}{4x^2}. \text{ Result: } 3/16.
\]

7. \( F(x) = kx \) so \( 20 = k \cdot 1 \) and \( k = 20 \). Thus, the work is \( \int_0^1 20x \, dx \). Result 10 lb ft.

8. \( \int_0^{\pi/4} (\cos x - \sin x) \, dx \). Result: \( \sqrt{2} - 1 \).

9. \( \frac{dx}{dy} = \frac{1}{2} (y^{1/2} - y^{-1/2}) \). The length is
\[
\int_1^9 \sqrt{1 + \frac{1}{4} (y^{1/2} - y^{-1/2})^2} \, dy = \int_1^9 \sqrt{\frac{1}{4} (y^{1/2} + y^{-1/2})^2} \, dy. \text{ Result: } 32/3.
\]

10. Average value: \( \frac{2}{\pi} \int_0^{\pi/2} \sin x \, dx \). Result: \( 2/\pi \).