Mixed Quantifiers with Negations

Relations with single quantifiers:

To clarify what follows, let’s start with Jones:

*Jones likes everyone!*

(For simplicity, let me use \( y \) as the variable for our universal quantifier.) Thus:

\[(y)Ljy\]

Easy as pie!

---With Negations:

Now let’s add a negation. This gives us two possibilities--before or after the quantifier.

First **consider the “~” coming before the quantifier:**

\[\neg(y)Ljy\]

This says

*It is not the case that Jones likes everyone, i.e., Jones doesn’t like every single person.*

But, by “reverse squiggle,” we know that

\[\neg(y)Ljy\]

is equivalent to

\[(\exists y)\neg Ljy\]

And this says

*There is someone who Jones doesn’t like.*

Second, **consider the “~” occurring after the quantifier.** Thus:

\[(y)\neg Ljy\]

This says

*Everyone is such that Jones doesn’t like that person, i.e., Jones doesn’t like anyone at all.*

Once again, by “reverse squiggle,” we know that

\[(y)\neg Ljy\]

is equivalent to

\[\neg(\exists y)Ljy.\]

And this says

*It is not the case that there is someone Jones likes, i.e., Jones likes no one at all.*

Of course, all of the above is to get our grounding before we consider multiple relations. So let’s move on!
Relations with Multiple Quantifiers:

Now, if all these statements are true of Jones, then they are all true of someone. So let’s replace all of the above examples with existential statements, using \( x \) as our variable. So, let us say that Someone likes everyone! This gives us:

\[(\exists x)(y)Lxy\]

Maybe not “easy as pie,” but not too tough.

--With Negations:

Once again, let’s add negations, in both of the variations we saw with Jones. First, consider what we get with the “~” before the universal quantifier:

\[(\exists x)\neg(y)Lxy\]

This says

\( \text{There is someone of whom it is false that they like every single person, i.e., There is someone who doesn’t like every single person.} \)

But remember that “\( \neg(y) \)” is equivalent to “(\( \exists y \))\neg.” And so

\[(\exists x)(\exists y)\neg Lxy\]

is equivalent to

\[(\exists x)(\exists y)\neg Lxy.\]

And this says

\( \text{There is someone who doesn’t like someone.} \)

Our second case is parallel to the second case above. So consider what we get with the “~” coming after the universal quantifier:

\[(\exists x)(y)\neg Lxy\]

This says

\( \text{There is someone for whom everyone is someone they don’t like, i.e., There is someone who doesn’t like anyone at all.} \)

Again, remember that “\( (y)\neg \)” is equivalent to “(\( \exists x \))\neg.” So

\[(\exists x)(y)\neg Lxy\]

is equivalent to

\[(\exists x)(\exists y)\neg Lxy.\]

And this says

\( \text{There is someone who likes no one at all.} \)
Finally, we wanted to translate the negations of all of these statements. I won’t talk through all the examples, but simply list them below.

\((\exists x)(\forall y)\neg Lxy\)

--Someone doesn’t like everyone
(i.e., they don’t like every single person).
--Someone doesn’t like someone.
--Not everyone likes everyone.

\((\exists x)(\forall y)\neg Lxy\)

--Someone doesn’t like anyone at all (doesn’t like even one person).
--There is someone who likes no one.

and their negations!

\(\neg(\exists x)(\forall y)Lxy\)

--No one doesn’t like everyone.
--Everyone likes someone.

\(\neg(\exists x)(\forall y)Lxy\)

--No one doesn’t like anyone at all.
--Everyone likes everyone.

Obviously, this list of equivalent ways to translate these formal statements is not complete. When we have wffs with multiple quantifiers and multiple negations, there are many possible variations. But this is exactly the same in “natural language.” The point of this entire exercise is to get us to think more carefully about what it means to talk about everything and something.

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