ECE 6950
Multirate Signal Processing
Analysis and Synthesis

Dr. Bradley J. Bazuin
Western Michigan University
College of Engineering and Applied Sciences
Department of Electrical and Computer Engineering
1903 W. Michigan Ave.
Kalamazoo MI, 49008-5329

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Polyphase Channelizer Performs Sample Rate Change Required for both Matched Filtering and Channel Frequency Spacing

**Downsampling**

![Downsampling Diagram](image1)

Figure 1. M-path Polyphase FDM-to TDM Channelizer.

**Upsampling**

![Upsampling Diagram](image2)

Figure 2. M-path Polyphase TDM-to FDM Channelizer.
Downsampling (Analysis)  
System Desired

- The system we consider is presented with 32 channels separated by 6-MHz center frequencies.
- The symbol rate of each channel is 5-MHz and each channel has been shaped by a sqrt Nyquist filter with 20% excess bandwidth.
- We select a 40-point IFFT to make available 8 channels to span the folded transition bands caused by the sampling process.
  - These channels are considered overhead channels and are discarded.
  - Use of the 40-point IFFT requires an input sample rate of 240-MHz to obtain the required 6-MHz channel spacing.
- In consideration of standard modem processing following the channelization we desire an output sample rate of 2-samples per symbol which is 10 MHz.
- Decimate by 24, 40 polyphase channels with 40 point IFFT.
Analysis Architecture

Figure 3. 40-Path Polyphase FDM-to TDM Channelizer with 24-to-1 Down-Sampling.
Upsampling (Synthesis)
System Desired

- The input symbol rate delivered to the shaping filter is 5 MHz.
- We select a 40-point IFFT to make available 8 channels to span the folded transition bands caused by the sampling process.
  - These channels are considered guard channels and are zero filled.
  - Use of the 40-point IFFT requires an output sample rate of 240-MHz to obtain the required 6-MHz output channel spacing.
Synthesis Architecture

Note: there is a required circular shift/phase rotation prior to the IFFT that is not shown.

Figure 5. Two Dimension Partition of Prototype Filter with Lowest Frequency Sinusoid of 40 Samples Extended to 48 Samples and Shifted in Stride of 48 and Phase Aligned by 8-Samples Shift.
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Filter banks in digital communications

- Communications channels and noise

**Fig. 1.** A simple model for a digital communication system. Here a sequence of symbols $x(n)$ is transmitted through a channel with transfer function $C(z)$ and additive noise $e(n)$.

**Fig. 4.** Relation between the input power spectrum $S_{xx}(e^{j\omega})$ and the effective noise power spectrum $S_{ee}(e^{j\omega})$ to achieve maximum rate for fixed total power. This is called the water filling rule.
Fig. 7. (a) The digital transmultiplexer, (b) operation of the interpolation filter $F_0(z)$, and (c) frequency responses of the transmitting filters (assumed to be ideal infinite order filters). Only the envelope of the samples of $f_0(n)$ are shown in (b).
Discrete Multitone Transmission
(Orthogonal Frequency Division Multiplexing, OFDM)

A symbol for every input

**Fig. 14.** DMT system based on the uniform DFT filter bank. The channel equalizer \(1/C(z)\) can be approximated well in many ways. An indirect but effective way to perform equalization is to introduce redundancies such as the cyclic prefix [13].
DMT or OFDM Advantages

- Symbols in different bins can be different
  - Adjust symbol type based on channel and channel noise. More noise, use no symbols or “simple” symbols or, for less noise, use “complex” symbols (more digital bits per symbol).

- Support multiple simultaneous communication paths
  - Each bin allocated to a different path

- Support variable data rates.
  - Multiple bins as needed for a communications path

- Signal processing has become possible
  - Multirate techniques and fast DSP processors

- Applicable for wired and wireless digital communications

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2-Channel QMF Bank

- Described for half-band filters in Harris text
- Can be generalized to M-channels using filter bank analysis followed by filter bank synthesis.
Mathematical Basis

\[ y_1(n) = \begin{cases} 
  x(n) * h_1(n) & n \text{ even} \\
  0 & n \text{ odd}
\end{cases} \quad (\text{equivalent to subsampling and interpolation by 2})
\]

\[ y_2(n) = \begin{cases} 
  x(n) * h_2(n) & n \text{ even} \\
  0 & n \text{ odd}
\end{cases} \quad (\text{equivalent to subsampling and interpolation by 2})
\]

where * denotes convolution. The signal \( y_1(n) \) and \( y_2(n) \) are, respectively, the outputs of the lower and upper frequency band. The reconstructed output signal from the inverse QMF filter bank is

\[ \hat{x}(n) = \hat{y}_1(n) * h_1(n) - \hat{y}_1(n) * h_2(n) = x(n) \]

The Design Problem

The flatness of \( H^\dagger(\omega) \) is the determining factor in the accuracy of the reconstruction of \( \hat{x}(n) \). If \( |H(\omega)|^2 + |H(\pi - \omega)|^2 = 1 \), reconstruction is perfect, otherwise, the reconstructed signal will be filtered by \( |H(\omega)|^2 + |H(\pi - \omega)|^2 = H_c(\omega) \). (\( e \) denoting error)
Design Constraints: Energy Criteria

Optimization Criteria

We next constructed a metric that expressed the two criteria as a single function of the filter characteristics. The following formulation was used.

$$E = E_r + \alpha E_s(f_{SB})$$  \hspace{1cm} (1)

where $E$ is the error criterion (objective function),

$$E_r = 2 \sum_{\omega=0}^{\pi/2} (H^2(\omega) + H^2(\pi - \omega) - 1)^2$$

$$E_s = \sum_{\omega=f_{SB}} H^2(\omega)$$

and

$$\alpha = \text{stop band weighting}$$

$$f_{SB} = \text{stop band edge}.$$

$E_r$ therefore corresponds to the ripple "energy", and $E_s$ is the out of band energy.

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M-channel maximally decimated quadrature mirror filters

\[
\hat{X}(z) = \frac{1}{M} \sum_{l=0}^{M-1} X(zW^{-l}) \sum_{k=0}^{M-1} H_k(zW^{-l}) F_k(z) \quad (1a)
\]

\[
\begin{bmatrix}
H_0(z) & H_1(z) & \cdots & H_{M-1}(z) \\
H_0(zW^{-1}) & H_1(zW^{-1}) & \cdots & H_{M-1}(zW^{-1}) \\
\vdots & \vdots & \ddots & \vdots \\
H_0(zW^{-M+1}) & H_1(zW^{-M+1}) & \cdots & H_{M-1}(zW^{-M+1}) \\
\end{bmatrix}
\begin{bmatrix}
F_0(z) \\
F_1(z) \\
\vdots \\
F_{M-1}(z)
\end{bmatrix}
= 
\begin{bmatrix}
T(z) \\
0 \\
\vdots \\
0
\end{bmatrix} \quad (1b)
\]
Project Notes

• Filter Generation
  – The initial filter can be almost anything
• Error computation for the “current” filter
  – If the error is not going down, the algorithm is not working
• A 2-channel filter can be “interpolated” to an M-channel filter
  – Sinc function (optimal frequency domain interpolation filter) convolution with the 2-channel filter.