ECE 6560
Multirate Signal Processing
Lecture 9

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Chapter 9: Polyphase Channelizers

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Digital Filter Bank Channelizer

• “Demodulation” of an entire set of channels simultaneously
  – Downconvert, filter, and decimate each of the channel bands.
  – Applications:
    • Multichannel radios
    • Spectrum Analyzer

Figure 6.3 Spectra Observed At Various Points in Processing Chain of Standard Down Converter
Polyphase FIR Review

• Notes from Chapter 6
  – A channelizer is based on performing multiple bandpass-filter polyphase FIR structures simultaneously!

  – The band-pass mixing (either from a center frequency to baseband or of the filter from baseband to the desired frequency) periodicity is related to the number of polyphase filter elements.

  – This relationship allows the use of a discrete Fourier transform (with all the related theories) to generate simultaneous outputs.
    • The DFT size is the same as the number of polyphase filter elements!
Channelizer

Figure 6.1 Spectrum of Multichannel Input Signal, Processing Task: Extract Complex Envelope of Selected Channel

Figure 6.2 Standard Single Channel Down Converter
Figure 6.3 Spectra

Spectra observed at various points in the processing chain of a standard radio receiver “Down Converter”
What about Mixing Prior to Filter Decimation

\[ y_k(m) = \sum_{n=-\infty}^{\infty} h(n) \cdot x(mM - n) \cdot \exp\left(-j2\pi \cdot \frac{(mM - n) \cdot k}{N}\right) \]

\[ y_k(m) = \sum_{n=-\infty}^{\infty} h(n) \cdot x(mM - n) \cdot \exp\left(-j2\pi \cdot m \cdot k \cdot \frac{M}{N}\right) \cdot \exp\left(j2\pi \cdot \frac{n \cdot k}{N}\right) \]

Let \( M = N \)

\[ y_k(m) = \sum_{n=-\infty}^{\infty} h(n) \cdot x(mM - n) \cdot \exp\left(j2\pi \cdot \frac{n \cdot k}{N}\right) \]
Mixing Continued

Let \( n = rM + \rho \)

\[
y_k(m) = \sum_{r=0}^{\lambda-1} \sum_{\rho=0}^{M-1} h(rM + \rho) \cdot x(mM - rM + \rho) \cdot \exp \left( j2\pi \cdot \frac{(rM + \rho) \cdot k}{M} \right)
\]

\[
y_k(m) = \sum_{r=0}^{\lambda-1} \sum_{\rho=0}^{M-1} h_{\rho}(r) \cdot \exp \left( j2\pi \cdot \frac{\rho \cdot k}{M} \right) \cdot x_{\rho}(m-r)
\]

\[
y_k(m) = \sum_{\rho=0}^{M-1} \exp \left( j2\pi \cdot \frac{\rho \cdot k}{M} \right) \cdot \sum_{r=0}^{\lambda-1} h_{\rho}(r) \cdot x_{\rho}(m-r)
\]

The complex weighting and summation of \( M \lambda \)-tap filters.

The computation is performed once every \( M \) input samples.
Simple Analysis Systems

- The previous analysis was for a simple “critically sampled” case with $M=K=N$.
  - May not be typical of desired communication system processing.
- ADC/input rates based on comm. bands and available devices.
  - “Magic numbers” used to define $K$ and “bin” spacings.
- Analysis output rates often based on data symbol or other embedded data signaling rates.
- Therefore, we need to allow “integer oversampling” and “rational fraction” oversampling.
  - $K/M$ may be a rational fraction not an integer!
Filter-Decimation Reformed

\[ X_k(m) = \sum_{l=\infty}^{\infty} h(l) \cdot x(mM - l) \cdot W_K^{(mM-l)(k+k_0)} \]  

(1)

where

\[ W_K^{nk} = \exp \left( -i2\pi nk/K \right) \]  

(2)

For a filter of length \( N\lambda \) where

\[
h(n) = \begin{cases} 
0 & n \leq 0 \\
h(n) & 0 < n \leq N\lambda \\
0 & N\lambda < n 
\end{cases} \]  

(3)

\[ K = N \quad l = rN - \rho \]  

(4)

\[
X_k(m) = W_N^{m(k+k_0)} \sum_{\rho=0}^{N-1} W_N^{\rho k_0} \sum_{r=1}^{N} [h(rN - \rho) \cdot W^{r-k_0}] \cdot x(mM - rN + \rho) \]  

(5)
Filter-Decimation (Polyphase-FFT)

Let

\[ k_0 = 0 \]  \hspace{1cm} (6)

\[ X_k(m) = W_M^{m(k+k_0)} \sum_{\rho=0}^{N-1} W_N^{\rho k} \left\{ \sum_{r=1}^2 h(rN - \rho) \cdot x(mM - rN + \rho) \right\} \]  \hspace{1cm} (7)

Cases:

1) Critically sampled, \( M = N \)

2) Integer ratio oversampling, \( M \cdot I = N \)

3) Arbitrary integer oversampling, \( Any \ M \)
Critical Sampling $N=M$

Let $M = N$

$$X_k(m) = \sum_{\rho=0}^{N-1} W_N^{\rho (k+k_n)} \left\{ \sum_{r=1}^{A} h(r N - \rho) \cdot x((m-r) \cdot N + \rho) \right\}$$  \hspace{1cm} (8)$$

$$X_k(m) = \sum_{\rho=0}^{N-1} W_N^{\rho r} \left\{ \sum_{r=1}^{A} h_{\rho}(r) \cdot x_{\rho}(m-r) \right\}$$  \hspace{1cm} (9)$$

where

$$h_{\rho}(r) = h(r N - \rho)$$

$$x_{\rho}(m-r) = x((m-r) \cdot N + \rho)$$  \hspace{1cm} (10)$$

Frequency channels are selected for $k = 0$ to $N - 1$
Text Based Representation

- The sum of the complex weighted polyphase elements

\[ y(nM,k) = \sum_{r=0}^{M-1} y_r(nM) e^{j \frac{2\pi r k}{M}} \]

**Figure 9.2** Polyphase Filter Bank as a Polyphase Filter Input and a DFT Output Process

TDM of M channels
Channel Bandwidths

- The channel bandwidths are based on the filter spectrum.

**Figure 9.3** Spectral Response of Two Filter Banks With Same Channel Spacing: One for Spectral Analysis and one for FDM Channel Separation.
Integer Oversampling N=M*I

Let

\[ X_k(m) = W_I^{mk} \sum_{\rho=0}^{N-1} W_N^{r\rho} \left\{ \sum_{r=1}^{\lambda} h(rN - \rho) \cdot x \left( m \frac{N}{I} - rN + \rho \right) \right\} \]  \hspace{1cm} (11)

\[ X_k(m) = W_I^{mk} \sum_{\rho=0}^{N-1} W_N^{r\rho} \left\{ \sum_{r=1}^{\lambda} h_\rho(r) \cdot x_\rho \left( m \frac{N}{I} - rN \right) \right\} \]  \hspace{1cm} (12)

where

\[ h_\rho(r) = h(rN - \rho) \]

\[ x_\rho \left( m \frac{N}{I} - rN \right) = x \left( m \frac{N}{I} - rN + \rho \right) \]  \hspace{1cm} (13)

Note how the x values are stepped or shifted by an amount N/I instead of N prior to each computation. In addition, a post-multiplication based on \( W_I^{mk} \) is required.
Fourier and Time Delay

- Think about the phase resulting from a pure time delay

\[ X_\delta(m) = \sum_{\rho=0}^{63} \delta(n-mM) W_N^{\rho k} \]

\[ X_\delta(m) = W_N^{mk} \]

- Then for M=32, N=64, I=2

\[ X_\delta(m) = W_2^{mk} = \exp(j \cdot \pi \cdot m \cdot k) = (-1)^{mk} \]

- If this “time delay” is applied as a circular shift in the data prior to the FFT, no post multiplication would be required!
The Fourier Matrix

\[
W_{nk}^N = \begin{bmatrix}
W_N^0 & W_N^0 & W_N^0 & \ldots & W_N^0 \\
W_N^0 & W_N^1 & W_N^2 & \ldots & W_N^{(N-1)} \\
W_N^0 & W_N^2 & W_N^4 & \ldots & W_N^{2(N-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
W_N^0 & W_N^{(N-1)} & W_N^{2(N-1)} & \ldots & W_N^{(N-1)(N-1)}
\end{bmatrix}
\]

- From the structure of the Fourier Matrix, here are only \( N \) unique “complex values” but they are repeated based on circular rotation around \( 2\pi n/N \).

\[
W_{nk}^N = \begin{bmatrix}
W_N^0 & W_N^0 & W_N^0 & \ldots & W_N^0 \\
W_N^0 & W_N^1 & W_N^2 & \ldots & W_N^{-1} \\
W_N^0 & W_N^2 & W_N^4 & \ldots & W_N^{-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
W_N^0 & W_N^{-1} & W_N^{-2} & \ldots & W_N^1
\end{bmatrix}
\]
FFT and post-Multiplication

• Focusing on the FFT and “Additional” Complex Multiplication

Focusing on

\[ X_k(m) = \sum_{\rho=0}^{63} W_{64}^{(m \cdot 32 + \rho)k} \cdot PP_{\rho}(m) \]

or more generally

\[ X_k(m) = \sum_{\rho=0}^{N-1} W_{N}^{(m \cdot M + \rho)k} \cdot PP_{\rho}(m) \]

Let

\[ \rho = (n - m \cdot M) \mod N \]  

(14)

\[ X_k(m) = \sum_{n=0}^{N-1} W_{N}^{k \cdot (m \cdot M + n \cdot -m \cdot M) \mod N} \cdot PP_{(n \cdot -m \cdot M) \mod N}(m) \]  

(15)

\[ X_k(m) = \sum_{n=0}^{N-1} W_{N}^{k \cdot n} \cdot PP_{(n \cdot -m \cdot M) \mod N}(m) \]  

(16)

Notice that this is the FFT of a circularly shifted input sequence! Therefore, to accomplish the equivalent of a post-FFT complex multiplication based on m, the FFT input sequence can be reordered prior to the transform.
Text Example: M=32, N=64, I=2

- Input data sequencing
  - Creating the polyphase filter elements prior to the Fourier Transform

\[ X_k(m) = W_2^{mk} \sum_{\rho=0}^{63} W_N^{\rho k} \left\{ \sum_{r=1}^{\lambda} h_\rho(r) \cdot x_\rho(m \cdot 32 - r \cdot 64) \right\} \]

\[ PP_\rho(m) = \sum_{r=1}^{\lambda} h_\rho(r) \cdot x_\rho(m \cdot 32 - r \cdot 64) \]

\[ X_k(m) = W_2^{mk} \sum_{\rho=0}^{63} W_N^{\rho k} \cdot PP_\rho(m) \]

Figure 9.4 Data Memory Loading for Successive 32-Point Sequences in a 64-Stage Polyphase Filter

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Potential Application 1

• FM radio stations are located every 200 kHz from from 88.1 MHz to 107.9 MHz.
  – 99 possible radio stations

• RF/IF design
  – Assume a 25.6 MHz band containing the 20 MHz bandwidth of the FM radio band.
  – We wish to complex “tune” 128 - 0.2 MHz radio stations consisting of 15 “dummy” or don’t care stations, 99 useful stations, and another 14 “dummy stations. In IF terms, 88.1 MHz => 15 x 0.2 kHz (3 MHz) or 98 MHz => 12.9 MHz.
  – The “downconverted” RF is now located from 2.9 to 22.9 MHz, centered at 12.9 MHz. (mixing frequencies of 85.1 MHz for low side or 110.9 MHz for high side)
  – Use K = 128 point transforms.

• Define M based on the desired sample rate for each radio channel.
  – To be conservative, lets have a “radio channel” complex sampling rate of 800 kHz where each output sample is a complex I,Q pair. Then, 25.6 MHz/0.8 MHz = 32 = M. (Note that we want M to be an integer.)

• Polyphase filter bandwidth desired: LPF passband 125 kHz stopband 375 kHz
  – Output sample rate supports “simple” digital demod methods
Potential Application 2

- Direct sampling of the AM radio Spectrum.
  - Sample rate 5.12 Msps real.
  - Bandpass filter before sampling: 500 kHz to 1800 kHz.
  - $K=512$ where we are interested in bins 56 (560 kHz) to 161 (1610 kHz).
  - For “easy” am demodulation, use a 40 kHz complex sample output rate. $M = 5120/40 = 128$.

- Polyphase filter desired: LPF 10 kHz passband 20 kHz stopband at 5120 kHz sample rate.

![Diagram of frequency spectrum]

Interpretation

• The sample rate $f_s$ and $K$ establish the “tuning frequencies” and tuning frequency step sizes for the “Nyquist zones” or channel bank bands.

• The sample rate and decimation rate, $M$, establish the bandwidth of the “Nyquist zones” or channel bank bands.

• If $K=M$, great, but that may not be the typical case!
  – adequate for narrowband signals of interest
  – adequate for determining “band” power
  – a useful design goal for system architecture
Arbitrary $M$

![Diagram of polyphase filter](image)

Where

\[
W^m_{k} = W^m_{N} \sum_{r=0}^{N-1} W^r_{N} \sum_{\rho=0}^{\hat{\lambda}} h(rN - \rho) \cdot x(mM - rN + \rho)
\] (7)

\[
X_k(m) = W^m_{N} \sum_{r=0}^{N-1} W^r_{N} \sum_{\rho=0}^{\hat{\lambda}} h_\rho(r) \cdot x_\rho(mM - rN)
\] (17)

where

\[
h_\rho(r) = h(rN - \rho)
\]

\[
x_\rho(mM - rN) = x(mM - rN + \rho)
\] (18)

The polyphase filter operates as expected with the input data shifted by $M$ for successive computations. As before, a post-multiplication is required, but is not based on $W^m_{N}$.  

Note: for some values of $k$ and $m$, no multiplication will be needed. For the remaining $k$ and $m$, the output is multiplied by a complex twiddle factor.
Arbitrary M FFT

Based on the previous results, we can form the following.

\[ PP_{\rho}(m) = \sum_{r=1}^{\rho} h_{\rho}(r) \cdot x_{\rho}(mM - rN) \]  \hspace{1cm} (19)

and

\[ X_k(m) = \sum_{\rho=0}^{N-1} W_N^{(mM + \rho)k} \cdot PP_{\rho}(m) \]

Again, let

\[ \rho = (n - m \cdot M)_{\text{mod}N} \]

First, the polyphase taps are formed from the M block updated samples for each m according to Equ. (19). The resulting polyphase filter outputs (PP) are then “circularly shifted” based on the modulo offset of mM, and finally Equ. (20) is computed.

\[ X_k(m) = \sum_{n=0}^{N-1} W_N^{k(mM + nM - m\cdot M)_{\text{mod}N}} \cdot PP_{(n-M)_{\text{mod}N}}(m) \]  \hspace{1cm} (20)

Note that N can define any desired channel spacing on 0 to 2\pi and that M defines any desired integer decimation of the input sample rate.
Example Problem from Text p. 234

• We have a signal containing 50 FDM channels separated by 192 kHz centers containing symbols modulated at 128 kHz by square-root Nyquist filters with 50% excess bandwidth. Our task is to baseband channelize all 50 channels and output data samples from each channel at 256 ks/s, which is two samples per symbol.

• The specifications of the process are listed next and the spectrum of the FDM input signal and of one of the 50 output signals is shown in Figure 9.6.
  - Number of Channels: 50
  - Channels Spacing: 192 kHz
  - Channel Symbol Rate: 128 kHz
  - Shaping Filter: SQRT Cosine Taper
  - Roll Off Factor $\alpha$: 50%
  - Output Sample Rate: 256 kHz (2x symbol rate)
Example Spectrum

- Input Spectrum
  - 50 channels spaced by 192 kHz
  - Passband and transition bands of outer channels
  - LPF Passband $\Rightarrow 49 \times 192 \text{ kHz} + 2 \times 64 \text{ kHz} = 9,536 \text{ kHz}$
  - LPF Stopband $\Rightarrow 49 \times 192 \text{ kHz} + 2 \times 64 \text{ kHz} + 2 \times 64 \text{ kHz} = 9,664 \text{ kHz}$ min.
  - Is this the optimal sample rate for “efficient” signal processing?
Sample Rate and Transform

- We start by selecting a transform size $N$ greater than the number of FDM channels to be processed. As indicated in (9.6), the product of the transform size and the channel spacing defines the input sample rate of the data collection process.

\[ f_s = N \cdot \Delta f \]  
(9.6)

- As shown in (9.7), a restatement of the Nyquist sampling criterion, the excess bandwidth spanned by the extra channels in the transform is allocated to the transition bandwidth of the analog anti-alias filter.

\[ f_s = 2\text{-Sided BW} + \text{Transition BW} \]
\[ 2\text{-Sided BW} = \# \text{Data Channels} \cdot \text{Channel Spacing} \]  
(9.7)
\[ \text{Transition BW} = \# \text{Nondata Channels} \cdot \text{Channel Spacing} \]
\[ \text{Transform Size} = \# \text{Data Channels} + \# \text{Nondata Channels} \]
Design Options

Table 9-1 List of Highly Composite Transform Sizes Considered for 50-Stage Polyphase Channelizer

<table>
<thead>
<tr>
<th>Transform Size N</th>
<th>Factors</th>
<th>Sample Rate (MHz)</th>
<th>Pass-Band Edge in FFT Bins</th>
<th>Stop-Band Edge in FFT Bins</th>
<th>Filter Order 0.1 dB Pass 60 dB Stop</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>2,3,3,3</td>
<td>10.368</td>
<td>25.5</td>
<td>28.5</td>
<td>9</td>
</tr>
<tr>
<td>60</td>
<td>3,4,5</td>
<td>11.520</td>
<td>25.5</td>
<td>34.5</td>
<td>7</td>
</tr>
<tr>
<td>64</td>
<td>2,2,2,2,2,2</td>
<td>12.288</td>
<td>25.5</td>
<td>38.5</td>
<td>6</td>
</tr>
<tr>
<td>72</td>
<td>2,2,2,3,3</td>
<td>13.824</td>
<td>25.5</td>
<td>46.5</td>
<td>6</td>
</tr>
<tr>
<td>80</td>
<td>2,2,2,2,5</td>
<td>15.360</td>
<td>25.5</td>
<td>54.5</td>
<td>5</td>
</tr>
</tbody>
</table>

- Important Factor: low-cost, efficient implementation
  - Radix-2 or $2^n$ FFT preferred!
  - Channel spacing $\times$ FFT size = 192 kHz $\times$ 64 = 12,288 kHz
  - Filter order

Decimation Rate

- Sample Rate = 12.288 MHz (64 x 192 kHz)
- Output Channel Rate = 256 kHz
  - 12,288 kHz/ 256 kHz = 48
  - A decimation rate of 48 must be applied

\[
x(n) \rightarrow \bigotimes \rightarrow h(n) \rightarrow \downarrow M \rightarrow X_k(m)
\]

\[
k_0 = 0 \quad K = N = 64 \quad M = 48
\]

\[
f_{\text{sin}} = 12.288 \text{MHz} \quad f_{\text{cout}} = 256 \text{kHz}
\]

\[
f_{\text{channel}} = 192 \text{kHz}
\]
Filter Bank Structures

- Maximally Decimated: 192 ksp output – incorrect
- Arbitrary M Decimation: 256 ksp output – correct
Polyphase Input Buffer

- Successive shifting and loading by blocks of 48 samples
- A circular queue or buffer is used. (memory addressing?)

\[
PP_{\rho}(m) = \sum_{r=1}^{\lambda} h_{\rho}(r) \cdot x_{\rho}(mM - rN)
\]
Polyphase Input Buffer

- Successive shifting and loading by blocks of 48 samples

\[
PP_\rho(m) = \sum_{r=1}^{\lambda} h_\rho(r) \cdot x_\rho(mM - rN)
\]

\[
X_k(m) = \sum_{n=0}^{N-1} W_N^{k\cdot n} \cdot PP_{(n-m\cdot M)_{\text{mod } N}}(m)
\]

Figure 9.8 Memory Contents for Successive 48-Point Input Data Blocks Into a 64-Point Prototype Pre-Polyphase Partitioned Filter and FFT

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Pre-FFT Cyclic Shifts

- A unique shift based on the least common denominator of $K$ and $N$ (16).

\[
X_k(m) = \sum_{n=0}^{N-1} W_n^k \cdot PP_{(n-m\cdot M)_{mod\,N}}(m)
\]
Implementation Options

- We have a signal containing 50 FDM channels separated by 192 kHz centers containing symbols modulated at 128 kHz by square-root Nyquist filters with 50% excess bandwidth. Our task is to baseband channelize all 50 channels and output data samples from each channel at 256 ks/s, which is two samples per symbol.

- The specifications of the process are listed next and the spectrum of the FDM input signal and of one of the 50 output signals is shown in Figure 9.6.
  - Number of Channels: 50
  - Channels Spacing: 192 kHz
  - Channel Symbol Rate: 128 kHz
  - Shaping Filter: SQRT Cosine Taper
  - Roll Off Factor $\alpha$: 50%
  - Output Sample Rate: 256 kHz (2x symbol rate)
Example Alternatives (p. 241)

- There are other ways to implement the previously defined problem. Section 9.2.1, p. 241, shows 5 ways.
  - Start at 12.288 Msps input
  - End at 256 ksps output
  - Apply “matched filter” (MF→square-root Nyquist)
    Use an arbitrary LPF when needed
  - Employ channel band filters and individual rational rate
    resampling to achieve the final rate
    - K=64, M=64 followed by 3-to-4 Rational Rate Change with MF
    - K=64, M=64 with MF followed by 3-to-4 Rational Rate Change
    - K=64, M=32 followed by 3-to-2 Rational Rate Change with MF
    - K=64, M=48 followed by MF
    - K=64, M=48 with MF
Option Block Diagrams

Design 1

Design 2

Design 3
Option Block Diagrams (2)

Design 4

Design 5

• Which option do you think is the most efficient?
  – The last one of course … proven in the following pages.
Design Option Descriptions

• Design 1. Channelize the FDM signal with 64-to-1 down sampling in the 64-path polyphase channelizing filter to obtain a per-channel output sample rate of 192 kHz, then up sample each channel by 4/3 to obtain the desired 256 kHz output sample rate, and finally pass each correctly sampled channel series through a matched filter. To reduce processing burden and cost, the matched filter is embedded in the 4/3-interpolator filter.

• Design 2. Channelize the FDM signal with 64-to-1 down sampling in the 64-path polyphase matched filter to obtain a per-channel output sample rate of 192 kHz, then up sample each matched filter series by 4/3 to obtain the desired 256 kHz output sample rate.

• Design 3. Channelize the FDM signal with a 32-to-1 down sampling in the 64-path polyphase channelizing filter to obtain a per-channel output sample rate of 384 kHz, then down sample by 3/2 to obtain the desired 256 kHz output sample rate, and finally pass each correctly sampled channel series through a matched filter. Here too, to reduce processing burden and cost, the matched filter is embedded in the 3/2-interpolator filter.

• Design 4. Channelize the FDM signal with a 48-to-1 down sampling in the 64-path polyphase channelizing filter to obtain a per-channel output sample rate of 256 kHz, and finally pass each correctly sampled channel series through a matched filter.

• Design 5. Channelize the FDM signal with a 48-to-1 down sampling in the 64-path polyphase matched filter to directly obtain a per-channel matched filter output sample rate of 256 kHz.
The specifications of the process are listed next and the spectrum of the FDM input signal and of one of the 50 output signals is shown in Figure 9.6.

- Number of Channels: 50
- Channels Spacing: 192 kHz
- Channel Symbol Rate: 128 kHz
- Shaping Filter: SQRT Cosine Taper
- Roll Off Factor $\alpha$: 50%
- Output Sample Rate: 256 kHz
M=64 Polyphase Filter

- “Pure Polyphase first stage”
  - $F_{\text{sin}} = 12,288$ kHz ($64 \times 192$ kHz)
  - $F_{\text{pass}} = 64$ kHz
  - $F_{\text{stop}} = (192-64=128)$ kHz possible, 96 kHz preferred

- $F_{\text{out}} = \text{based on design (192, NA, 384, 256, NA)}$
Matched Filter Rational Rate Change

- The matched filter is a Square-root Nyquist filter.
  - Input Sample Rate: 768 kHz (varies with design 192 x 4)
  - Channel Symbol Rate: 128 kHz
  - Shaping Filter: SQRT Cosine Taper
  - Roll Off Factor $\alpha$: 50%
  - Output Sample Rate: 256 kHz

- From NyquistTestv2.m (k=5 for option 1)
  - $hsqnyq$ = $\text{firrcos}(2*k*M,fsymbol/2,alpha,fsample,'rolloff','sqrt')$;
  - $hsqnyqfh$ = $\text{nyq_fharris}(fsymbol,fsample,alpha,k,1)$;
Option 1

Design 1. Channelize the FDM signal with 64-to-1 down sampling in the 64-path polyphase channelizing filter to obtain a per-channel output sample rate of 192 kHz, then up sample each channel by 4/3 to obtain the desired 256 kHz output sample rate, and finally pass each correctly sampled channel series through a matched filter. To reduce processing burden and cost, the matched filter is embedded in the 4/3-interpolator filter.
Option 2

- Channelize the FDM signal with 64-to-1 down sampling in the 64-path polyphase matched filter to obtain a per-channel output sample rate of 192 kHz, then up sample each matched filter series by 4/3 to obtain the desired 256 kHz output sample rate.
Option 3

- Design 3. Channelize the FDM signal with a 32-to-1 down sampling in the 64-path polyphase channelizing filter to obtain a per-channel output sample rate of 384 kHz, then down sample by 3/2 to obtain the desired 256 kHz output sample rate, and finally pass each correctly sampled channel series through a matched filter. Here too, to reduce processing burden and cost, the matched filter is embedded in the 3/2-interpolator filter.
Option 4

- Design 4. Channelize the FDM signal with a 48-to-1 down sampling in the 64-path polyphase channelizing filter to obtain a per-channel output sample rate of 256 kHz, and finally pass each correctly sampled channel series through a matched filter.
Option 5

- Design 5. Channelize the FDM signal with a 48-to-1 down sampling in the 64-path polyphase matched filter to directly obtain a per-channel matched filter output sample rate of 256 kHz.
Option Comparison

<table>
<thead>
<tr>
<th>Design Option</th>
<th>Ops/Channel</th>
<th>Relative Efficiency wrt Direct Conversion</th>
<th>Relative Efficiency wrt 48-to-1 Polyphase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Down Conversion</td>
<td>1384</td>
<td>100 %</td>
<td>25.8</td>
</tr>
<tr>
<td>64-to-1 Polyphase Channelize &amp; 3-to-4 MF</td>
<td>68.2</td>
<td>4.93 %</td>
<td>1.27</td>
</tr>
<tr>
<td>64-to-1 Polyphase MF &amp; 3-to-4 Interpolate</td>
<td>71.6</td>
<td>5.17 %</td>
<td>1.33</td>
</tr>
<tr>
<td>32-to-1 Polyphase MF &amp; 3-to-2 MF</td>
<td>68.3</td>
<td>4.93 %</td>
<td>1.27</td>
</tr>
<tr>
<td>48-to-1 Polyphase Channelizer &amp; MF</td>
<td>78.4</td>
<td>5.66 %</td>
<td>1.46</td>
</tr>
<tr>
<td>48-to-1 Polyphase MF</td>
<td>53.7</td>
<td>3.88 %</td>
<td>1.00</td>
</tr>
</tbody>
</table>

- This is an example of tasks that DSP system engineer and designer must perform to “optimize” implementations (H/W or S/W) and algorithm applications.
Technical Paper Review
