Undergrad, I had 2 advanced PHYS courses back to back – with opposing forms of spherical coordinates!

The worst semester of my life as an undergrad!

But what works for the cylinder, doesn’t quite work the same way for the sphere. Read on, gentle physics student!

**For a Sphere, we need Spherical Coordinates**

Let \( \rho = \frac{m}{V} \), so \( dm = \rho dV \), which for cylindrical coordinates is \( dV = \rho r dr d\theta dz \). Since \( \rho \) represents the radial variable, and \( z \) is also the distance from the axis of rotation, we can just multiply \( r \) in:

\[
I = \int r^2 dm
\]

But the \( z \) is parallel to the axis of rotation, and so nothing in the \( z \)-direction is going to change the distance to the axis of rotation, so we could choose to think of \( dm \) as little vertical slices around the cylinder, so that we are really looking at a mass/area.

Instead, we should consider that distance from the axis of rotation is not \( r \) but \( r \sin \phi \). (Pointing straight up the \( z \)-axis is \( \phi = 0 \) and down the \( z \)-axis is \( \phi = \pi \)).

Now as tempting as it is to say that we are done and should just integrate to get:

\[
I = \int r^2 \sin \phi \rho r dr d\theta d\phi = \rho \int r^2 d\phi \int r dr \int \sin \phi d\theta = \rho \int \sin \phi \bigg( \frac{r^4}{4} \bigg) \bigg( 2 \pi r \bigg) \bigg( \frac{1}{2} r^2 \bigg) = \frac{\pi \rho R^6}{6}
\]

This is neither the right physics nor the right answer.

Note that all three of these methods yield the same answer for the moment of inertia of the cylinder.

But what works for the cylinder, doesn’t quite work the same way for the sphere. Read on, gentle physics student!

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1 The Ugly Truth They Didn’t Want To Warn You Of: For reasons that have never made sense to Dr. Phil, there are **two** competing versions of Spherical Coordinates, which interchange \( \theta \) and \( \phi \). The worst semester of my life as an undergrad, I had 2 advanced PHYS courses back to back – with opposing forms of spherical coordinates!