ME 3600 Control Systems
Design Problem 4.2 – Roll Control of a Small Aircraft

The block diagram of the roll control system is shown below.

The variables $\theta_d(s)$ and $\theta(s)$ represent the desired and actual roll angles of the aircraft, and the variable $T_d(s)$ represents a disturbance roll torque on the aircraft. The problem statement indicates that we are to find reasonable values of the product $KK_1$ so the system maintains small roll angles in the presence of a disturbance, and that a desirable transient response is achieved.

Given the block diagram, the closed-loop and disturbance transfer functions are

$$\frac{\theta(s)}{\theta_d(s)} = \frac{KK_1}{s^3 + 4s^2 + 9s + KK_1}$$

$$\frac{\theta(s)}{T_d(s)} = \frac{s}{s^3 + 4s^2 + 9s + KK_1}$$

From these transfer functions it is clear that the system has no steady-state error due to a step command in the desired roll angle, and that there is no steady-state error due to a step input disturbance. The step responses of these transfer functions for various $KK_1$ values are shown below.
Steady-State Error Due to a Ramp Input

The steady state error of the system due to a ramp input may be calculated using the final value theorem. Using the block diagram, the error transfer function is

\[
\frac{E(s)}{\theta_d(s)} = \frac{s(s^2 + 4s + 9)}{s^3 + 4s^2 + 9s + KK_1}
\]

Using the error transfer function, the steady-state error due to a ramp input is

\[
e_{ss} = \lim_{s \to 0} \left( \frac{1}{s^2} \cdot \frac{E(s)}{\theta_d(s)} \right) = \lim_{s \to 0} \left( \frac{1}{s^2} \cdot \frac{s(s^2 + 4s + 9)}{s^3 + 4s^2 + 9s + KK_1} \right) = \frac{9}{KK_1}
\]

The roll ramp response of the system is shown in the figure below for various \(KK_1\) values.