**Example System: Simple Speed Control System**

Consider a car of mass \( m \) traveling along a road with wind resistance (proportional to the speed of the car) as shown in the diagram. Applying Newton’s 2\(^{nd} \) law in the direction of travel and neglecting friction, we can write

\[
\sum F = f(t) - cv = m\dot{v}
\]

or

\[
m\dot{v} + cv = f(t)
\]

Using Laplace transforms, the transfer function for the system is

\[
\frac{V}{F(s)} = \frac{1/m}{s + c/m}.
\]  \( \text{(1)} \)

**Open-Loop, Proportional Speed Control**

Now consider *proportional, open-loop speed control* of the car as indicated in the block diagram. The system transfer function is

\[
\frac{V}{P(s)} = \frac{K/m}{s + c/m}
\]  \( \text{(2)} \)

The final value due to a unit step input \( P(s) = 1/s \) is

\[
V_{ss} = (K/m)/(c/m) = K/c.
\]

Fig. 1 shows the step response of this system for \( K = 300, 600, \) and \( 900 \) using the parameters shown in Eq. (3). Note that the value of \( K \) affects the magnitude of the response, but it does not affect how long the car takes to reach a new final speed.

\[
m = 100 \text{ slugs} \]
\[
c = 20 \text{ (lb-s/ft)}
\]  \( \text{(3)} \)
Closed-Loop, Proportional Speed Control

Finally, consider the proportional, closed-loop speed control of the car as indicated in the block diagram. The transfer function of this system is

\[
\frac{V}{V_d}(s) = \frac{K/m}{s + c/m + K/m} = \frac{K/m}{s + (c + K)/m}
\]

The final value due to a step input \( V_d(s) = 1/s \) is

\[
v_{ss} = \frac{K}{m} \frac{1}{(c + K)/m} = \frac{K}{c + K}
\]

Fig. 2 shows the step response of the system for \( K = 300, 600, \) and 900. Note that the value of \( K \) affects both the magnitude of the response and the time it takes the car to reach a new final speed.
In theory, the value of $K$ could be \textit{increased} further to make the steady state response ($v_{ss}$) closer to the commanded value ($= 1 \text{ (ft/s)}$) and the settling time smaller and smaller. However, as these changes are made, the force required to move the car becomes higher and higher. Fig. 3 shows the driving force $f(t)$ associated with the \textit{unit step responses} in Fig. 2. Clearly, higher velocity commands and higher gains will cause the forces to eventually become unrealistic.