ME 459 Dynamics of Machinery
Introduction to Lagrangian Dynamics

Newton/Euler Equations of Motion

One approach to finding the equations of motion (EOM) of a mechanical system is to use the Newton/Euler equations of motion

\[ \sum_i F_i = ma_G \]
\[ \sum_i (M_G)_i = \sum_i (r_i \times F_i) = I_G \alpha \]

or

\[ \sum_i F_i = ma_G \]
\[ \sum_i (M_P)_i = \sum_i (p_i \times F_i) = I_G \alpha + (r_{GP} \times ma_G) \]

where \( G \) is the mass center of the body and \( P \) is any point. In this approach, bodies are isolated one-by-one using free body diagrams. Then the above equations are used to write the EOM. As a result, these equations contain unknown constraint forces and moments. Hence, the equations form a set of differential/algebraic equations.

Lagrange's Equations of Motion

The application of Lagrange's equations of motion differs from the application of the Newton/Euler EOM in the following ways:

- Focus is on the entire system rather than individual components.
- EOM are formulated in terms of the scalar functions of work and kinetic energy.
- Constraint forces and moments that do no work are eliminated from the analysis.

For many systems the resulting EOM form a set of differential equations. For more complex systems the EOM form a set of differential/algebraic equations. The specific form of Lagrange's equations will be presented later.

Note: Lagrange's equations are not the only system-based formulation of EOM. Other system-based formulations include D'Alembert's Principle and Kane's Equations.