1. Consider the following steady, incompressible, two–dimensional velocity field:

\[ \mathbf{V} = (u, v) = (0.5 + 1.2x)\hat{i} + (-2.0 - 1.2y)\hat{j} \]

Generate an analytical expression for the flow streamlines.

2. A steady, incompressible, two–dimensional velocity field is given by:

\[ \mathbf{V} = (u, v) = (1 + 2.5x + y)\hat{i} + (-0.5 - 1.5x - 2.5y)\hat{j} \]

where the \( x \)– and \( y \)–coordinates are in m and the magnitude of the velocity is in m/s.

a) Determine if there are any stagnation points in this flow field, and if so, where are they.

b) Calculate the material acceleration at the point \((x = 2 \text{ m}, y = 3 \text{ m})\).

3. A general equation for a steady, two–dimensional velocity field that is linear in both spatial directions \((x \text{ and } y)\) is:

\[ \mathbf{V} = (u, v) = (U + a_1x + b_1y)\hat{i} + (V + a_2x + b_2y)\hat{j} \]

where \( U \) and \( V \) and the coefficients are constants. Their dimensions are assumed to be appropriately defined.

a) Calculate the \( x \)– and \( y \)–components of the acceleration field.

b) Calculate the linear strain rates in the \( x \)– and \( y \)–directions.

c) Calculate the shear strain rate in the \( xy \)–plane.

d) Calculate the vorticity vector. In which direction does the vorticity vector point?

4. Consider the following steady, three – dimensional velocity field:

\[ \mathbf{V} = (u, v, w) = (3 + 2x - y)\hat{i} + (2x - 2y)\hat{j} + (0.5xy)\hat{k} \]

Determine the vorticity vector as a function of space \((x, y, z)\).

5. Consider fully developed Couette Flow –flow between two infinite parallel plates separated by a distance \( h \), with the top plate moving and the bottom plate stationary. The flow is steady, incompressible, and two–dimensional in the \( xy \)–plane. The velocity field is given by

\[ \mathbf{V} = (u, v) = \left( V \frac{Y}{h} \right)\hat{i} + 0\hat{j} \]

Is the flow rotational or irrotational? If it is rotational, calculate the vorticity component in the \( z \)–direction. Do fluid particles in this flow rotate clockwise or counterclockwise?
6. Consider the fully developed two–dimensional Poiseuille flow –flow between two infinite parallel plates separated by a distance \( h \), with both the top and bottom plates stationary, and a forced pressure gradient (constant and negative) \( \frac{dP}{dx} \) driving the flow as shown.

The flow is steady, incompressible, and two–dimensional in the \( xy \)–plane. The velocity components are given by

\[
\begin{align*}
  u &= \frac{1}{2\mu} \frac{dP}{dx} (y^2 - hy) \\
  v &= 0
\end{align*}
\]

where \( \mu \) is the fluid’s viscosity. Is this flow rotational or irrotational? If it is rotational, calculate the vorticity component in the \( z \)–direction. Do fluid particles in this flow rotate clockwise or counterclockwise?

7. Air enters a nozzle steadily at 2.21 kg/m\(^3\) and 30 m/s and leaves at 0.762 kg/m\(^3\) and 180 m/s. If the inlet area of the nozzle is 80 cm\(^2\), determine (a) the mass flow rate through the nozzle, and (b) the exit area of the nozzle.

8. A 1 m\(^3\) rigid tank initially contains air whose density is 1.18 kg/m\(^3\). The tank is connected to a high pressure supply line through a valve. The valve is opened, and air is allowed to enter the tank until the density in the tank rises to 7.2 kg/m\(^3\). Determine the mass of air that has entered the tank.

9. Air whose density is 0.078 lbm/ft\(^3\) enters the duct of an air conditioning system at a volume flow rate of 450 ft\(^3\)/min. If the diameter of the duct is 10 in, determine the velocity of the air at the duct inlet and the mass flow rate of air.

10. A horizontal water jet of constant velocity \( V \) impinges normally on a vertical flat plate and splashes off the sides in the vertical plane. The plate is moving towards the oncoming water jet with velocity \( \frac{1}{2} V \). If a force \( F \) is required to maintain the plate stationary, how much force is required to move the plate towards the jet?
11. A 100 ft$^3$/s water jet is moving in the positive $x$-direction at 20 ft/s. The stream hits a stationary splitter, such that half of the flow is diverted upward at 45$^\circ$ and the other half is directed downward, and both streams have a final speed of 20 m/s. Disregarding the gravitational effects, determine the $x$- and $z$-components of the force required to hold the splitter in place against the water force.

![Diagram of water stream hitting splitter](image)

12. The water level in a tank is 66 ft above the ground. A hose is connected to the bottom of the tank at the ground level and the nozzle at the end of the hose is pointed straight up. The tank cover is airtight, but the pressure over the water surface is unknown. Determine the minimum tank air pressure (gage) that will cause a water stream from the nozzle to rise 90 ft from the ground.

13. A pressurized 2 m diameter tank of water has a 10 cm diameter orifice at the bottom, where water discharges to the atmosphere. The water level initially is 3 m above the outlet. The tank pressure above the water level is maintained at 450 kPa absolute and the atmospheric pressure is 100 kPa. Neglecting the frictional effects, determine (a) how long it will take for half of the water in the tank to be discharged and (b) the water level in the tank after 10 s.

14. A wind tunnel draws atmospheric air at 20°C and 101.3 kPa by a large fan located near the exit of the tunnel. If the air velocity in the tunnel is 80 m/s, determine the pressure in the tunnel.

![Diagram of wind tunnel](image)

15. A turbine is designed to extract energy from a water source flowing through a 10 cm diameter pipe at a pressure of 800 kPa with an average velocity of 10 m/s. If the turbine is 90 percent efficient, how much energy can be produced if the water is emitted to the atmosphere through a 20 cm diameter pipe?
16. Air enters a compressor at 25°C and 10 kPa with negligible velocity. It exists through a 2 cm diameter pipe at 400 kPa and 160°C with a velocity of 200 m/s. Determine the heat transfer if the power required is 18 kW.

17. A 90 percent efficient turbine accepts water at 400 kPa in a 16 cm diameter pipe. What is the maximum power output if the flow rate is (a) 0.08 m³/s, (b) 0.06 m³/s, and (c) 0.04 m³/s? the water is emitted to the atmosphere.

18. Consider the steady–two dimensional incompressible velocity field given by:
   \[ \mathbf{V} = (u, v) = (1.3 + 2.8x)i + (1.5 - 2.8y)j \]
   Verify that the flow is incompressible.

19. Consider the following steady, two–dimensional incompressible velocity field:
   \[ \mathbf{V} = (u, v) = (ax + b)i + (-ay + cx)j \]
   Is this flow irrotational? If so, generate an expression for the velocity potential function.

20. Consider a steady, two–dimensional incompressible, irrotational velocity field specified by its potential function:
   \[ \Phi = 5(x^2 - y^2) + 2x - 4y \]
   (a) Calculate the velocity components \( u \) and \( v \). (b) Verify that the velocity field is irrotational in the region in which \( \Phi \) applies. (c) Generate an expression for the stream function in this region.

21. Consider a steady, incompressible, parallel, laminar flow of a viscous fluid falling between two infinite vertical walls. The distance between the walls is \( h \), and gravity acts negative to the \( z \)-direction. There is no applied (force) pressure driving the flow–the fluid falls by gravity alone. The pressure is constant everywhere in the flow field. Calculate the velocity field.
22. Consider steady, incompressible, laminar flow of a Newtonian fluid in an infinitely long round pipe annulus of inner radius $R_i$ and outer radius $R_o$. Ignore the effects of gravity. A constant negative pressure gradient $\partial P/\partial z$ is applied in the $z$–direction, $(\partial P/\partial z) = (P_2 - P_1) / (z_2 - z_1)$, where $z_1$ and $z_2$ are two arbitrary locations along the $z$–axis, and $P_1$ and $P_2$ are the pressures at those locations. The pressure gradient may be caused by a pump. Derive an expression for the velocity field in the annular space in the pipe.

![Diagram](image.png)

23. The stream function for steady, incompressible, two–dimensional flow over a circular cylinder of radius $a$ and free stream velocity $U$ is $\Psi = V_\infty \sin \theta (r - a^2/r)$ for the case in which the flow is approximated as irrotational. Generate an expression for the velocity potential function $\Phi$ for this flow as a function of $r$ and $\theta$, and parameters $V_\infty$ and $a$.

![Diagram](image.png)

24. A source and a sink of equal strength of 20 ft$^3$/s are located as shown in the figure. What is the flow velocity at (15,15)

![Diagram](image.png)

25. The potential flow about a cylinder of radius 2 ft where the free stream velocity is 10 ft/s needs to be represented. What should the stream function be? What is the drop in pressure at the top of the cylinder from the free stream pressure at infinity? What is the increase of pressure above the free stream pressure at the stagnation point? Assume the fluid flowing over the cylinder is air ($\rho = 0.002378$ slug/ft$^3$)