Thermoelectric Generators

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1. Thermoelectric generator

1.1 Basic Equations

In 1821, Thomas J. Seebeck discovered that an electromotive force or potential difference could be produced by a circuit made from two dissimilar wires when one junction was heated [1]. This is called the *Seebeck effect*. In 1834, Jean Peltier discovered the reverse process that the passage of an electric current through a thermocouple produces heating or cooling depended on its direction [2]. This is called the *Peltier effect* (or *Peltier cooling*). In 1854, William Thomson discovered that if a temperature difference exists between any two points of a current-carrying conductor, heat is either absorbed or liberated depending on the direction of current and material [3]. This is called the *Thomson effect* (or *Thomson heat*). These three effects are called the *thermoelectric effects*.

The mechanisms of thermoelectricity were not understood well until the discovery of electrons in the end of the 19th century. Now it is known that solar energy, an electric field, or thermal energy can liberate some electrons from their atomic binding even at room temperature (from the valence band to the conduction band of a conductor) where the electrons become free to move randomly. However, when a temperature difference across a conductor is applied as shown in Figure 1, the hot region of the conductor produces more free electrons and diffusion of the electrons (charge carriers including holes) naturally occurs from the hot region to the cold region. An electromotive force (*emf*) is generated in a way that an electric current flows against the temperature gradient.

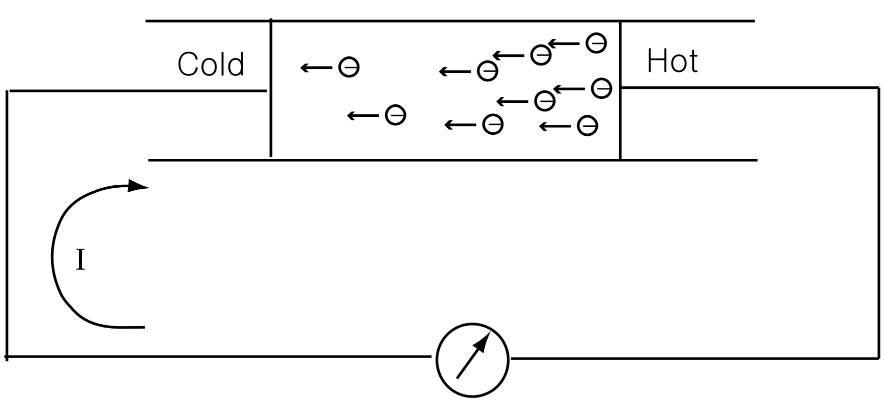
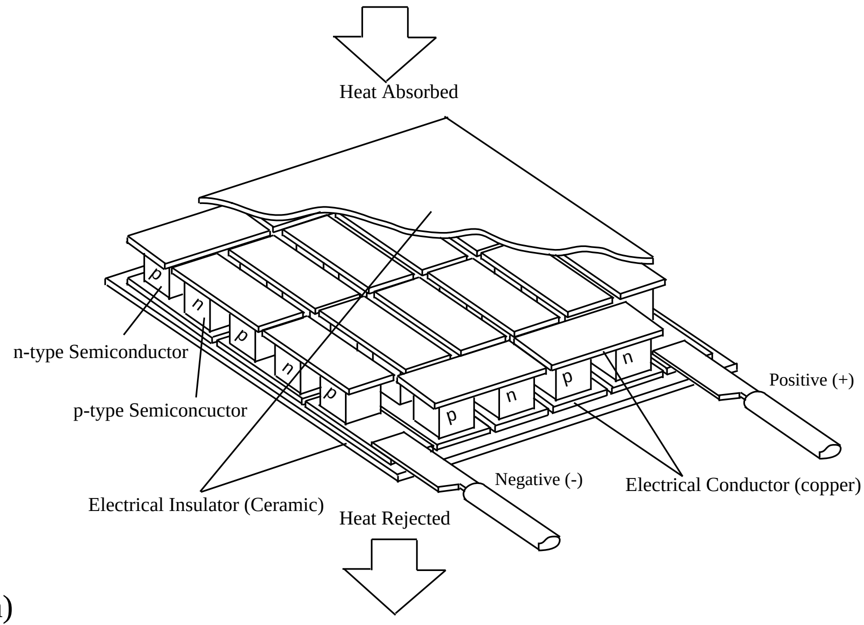
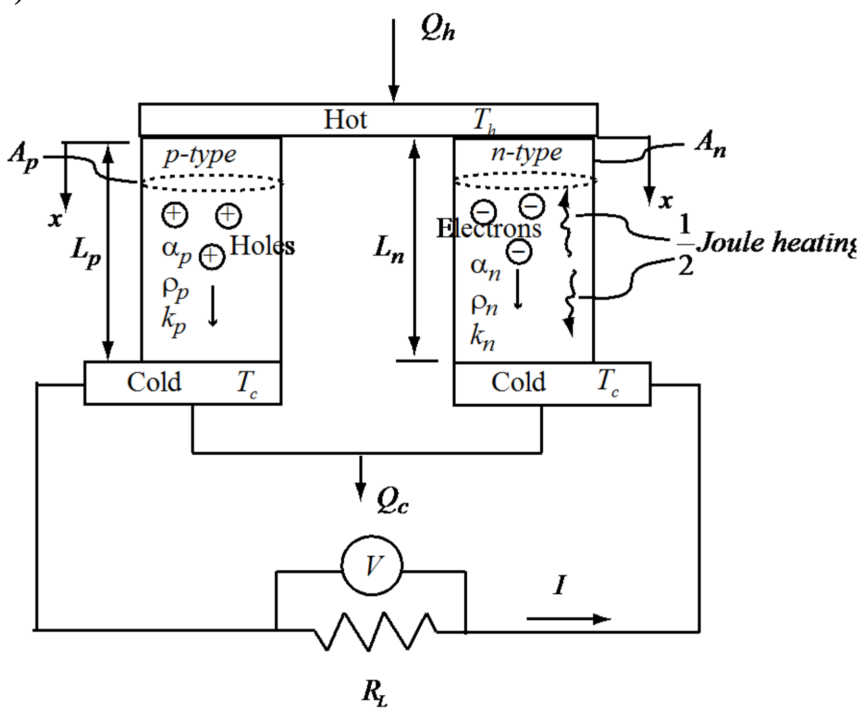


Figure 1 Electron concentration in a thermoelectric material.

A large number of thermocouples, each of which consists of p-type and n-type semiconductor elements, are connected electrically in series and thermally in parallel by sandwiching them between two high thermal conductivity but low electrical conductivity ceramic plates to form a module, which is shown in Figure 2a.



(a)



(b)

Figure 2. (a) Cutaway of a thermoelectric generator module, and (b) a *p*-type and *n*-type thermocouple.

Consider a steady-state one-dimensional thermoelectric generator module as shown in Figure 2a. The module consists of many *p*-type and *n*-type thermocouples in Figure 2b. We assume that the electrical and thermal contact resistances are negligible, the Seebeck coefficient is independent of temperature, and the radiation and convection at the surfaces of the elements are negligible.

The heat absorbed at the hot junction of temperature T_h is expressed as

$$\dot{Q}_h = n\left[\alpha T_h I - \frac{1}{2} I^2 R + K\left(T_h - T_c\right)\right] \tag{1}$$

where

$$\alpha = \alpha_p - \alpha_n \tag{2}$$

$$R = \frac{\rho_p L_p}{A_p} + \frac{\rho_n L_n}{A_n} \tag{3}$$

$$K = \frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n} \tag{4}$$

The first term in Equation (1) is the Seebeck effect or called the Peltier heat, the second term is half of the Joule heating, and the last term is the thermal conduction. If we assume that *p*-type and *n*-type thermocouples are similar ($L_p = L_n$ and $A_p = A_n$), we have that $R = \rho L/A$ and $K = kA/L$, where $\rho = \rho_p + \rho_n$ and $k = k_p + k_n$. Equation (1) is called the *ideal equation* which has been widely used in science and industry. The rate of heat liberated at the cold junction is given by

$$\dot{Q}_c = n\left[\alpha T_c I + \frac{1}{2} I^2 R + K\left(T_h - T_c\right)\right] \tag{5}$$

From the 1st law of thermodynamics across the thermoelectric module, which is $\dot{W}_n = \dot{Q}_h - \dot{Q}_c$. The power output is then expressed in terms of the internal properties as

$$\dot{W}_n = n\left[\alpha I\left(T_h - T_c\right) - I^2 R\right] \tag{6}$$

However, the power output in Figure 2b can be defined by an external load resistance as

$$\dot{W}_n = n I^2 R_L \tag{7}$$

Equating Equations (6) and (7) with $\dot{W}_n = I V_n$ gives the voltage as

$$V_n = nIR_L = n\left[\alpha\left(T_h - T_c\right) - IR\right] \tag{8}$$

1.2 Performance Parameters of a Thermoelectric Module

From Equation (8), the electrical current for the module is obtained as

$$I = \frac{\alpha\left(T_h - T_c\right)}{R_L + R} \tag{9}$$

Note that the current I is independent of the number of thermocouples. Inserting this into Equation (8) gives the voltage across the module by

$$V_n = \frac{n\alpha\left(T_h - T_c\right)}{\frac{R_L}{R} + 1}\left(\frac{R_L}{R}\right) \tag{10}$$

Inserting Equation (9) in Equation (7) gives the power output as

$$\dot{W}_n = \frac{n\alpha^2\left(T_h - T_c\right)^2}{R}\frac{\frac{R_L}{R}}{\left(1 + \frac{R_L}{R}\right)^2} \tag{11}$$

The thermal (or conversion) efficiency is defined as the ratio of the power output to the heat absorbed at the hot junction:

$$\eta_{th} = \frac{\dot{W}_n}{\dot{Q}_h} \tag{12}$$

Inserting Equations (1) and (11) into Equation (12) gives an expression for the thermal efficiency:

$$\eta_{th} = \frac{\left(1 - \frac{T_c}{T_h}\right)\frac{R_L}{R}}{\left(1 + \frac{R_L}{R}\right) - \frac{1}{2}\left(1 - \frac{T_c}{T_h}\right) + \frac{\left(1 + \frac{R_L}{R}\right)^2 \frac{T_c}{T_h}}{ZT_c}} \tag{13}$$

where $Z = \frac{\alpha^2}{\rho k}$ or, equivalently, $Z = \frac{\alpha^2}{RK}$, which is called the figure of merit, presenting the efficacy of the material: the higher the figure of merit, the better the performance of the thermoelectric generator.

1.3 Maximum Parameters for a Thermoelectric Generator Module

Since the maximum current inherently occurs at the short circuit where $R_L = 0$ in Equation (9), the maximum current for the module is

$$I_{\max} = \frac{\alpha (T_h - T_c)}{R} \tag{14}$$

The maximum voltage inherently occurs at the open circuit, where I = 0 in Equation (8). The maximum voltage is

$$V_{\max} = n\alpha (T_h - T_c) \tag{15}$$

The maximum power output is attained by differentiating the power output $\dot{W}$ in Equation (11) with respect to the ratio of the load resistance to the internal resistance and setting it to zero. The result yields a relationship of $R_L / R = 1$, which leads to the maximum power output as

$$\dot{W}_{\max} = \frac{n\alpha^2 (T_h - T_c)^2}{4R} \tag{16}$$

The maximum conversion efficiency can be obtained by differentiating the thermal efficiency in Equation (13) with respect to the ratio of the load resistance to the internal resistance and setting it to zero. The result yields a relationship of $R_L / R = \sqrt{1 + Z\overline{T}}$. Then, the maximum conversion efficiency $\eta_{\max}$ is

$$\eta_{\max} = \left(1 - \frac{T_c}{T_h}\right) \frac{\sqrt{1 + Z\overline{T}} - 1}{\sqrt{1 + Z\overline{T}} + \frac{T_c}{T_h}} \tag{17}$$

$\overline{T}$ is the average temperature of T_c and T_h. On the basis of T_c, $Z\overline{T}$ is expressed by

$$Z\overline{T} = \frac{ZT_c}{2}\left(1 + \left(\frac{T_c}{T_h}\right)^{-1}\right) \tag{18}$$

There are so far four intrinsic maximum parameters, which are $I_{\max}$, $V_{\max}$, $\dot{W}_{\max}$, and $\eta_{\max}$. However, there is also the maximum power efficiency. Most manufacturers have been using the maximum power efficiency as a specification for their products. The maximum power efficiency is obtained by letting $R_L/R=1$ in Equation (13). The maximum power efficiency η_{mp} is

$$\eta_{mp}=\frac{1-\dfrac{T_c}{T_h}}{2-\dfrac{1}{2}\left(1-\dfrac{T_c}{T_h}\right)+\dfrac{4\dfrac{T_c}{T_h}}{ZT_c}} \tag{19}$$

Note that there are two thermal efficiencies: the maximum power efficiency η_{mp} and the maximum conversion efficiency $\eta_{\max}$.

1.4 Normalized Parameters

If we divide the active values by the maximum values, we can normalize the characteristics of a thermoelectric generator. The normalized power output can be obtained by dividing Equation (11) by Equation (16), which is

$$\frac{\dot{W}}{\dot{W}_{\max}}=\frac{4\dfrac{R_L}{R}}{\left(\dfrac{R_L}{R}+1\right)^2} \tag{20}$$

Equations (9) and (14) give the normalized currents as

$$\frac{I}{I_{\max}}=\frac{1}{\dfrac{R_L}{R}+1} \tag{21}$$

Equations (10) and (15) give the normalized voltage as

$$\frac{V_n}{V_{\max}}=\frac{\dfrac{R_L}{R}}{\dfrac{R_L}{R}+1} \tag{22}$$

Equations (13) and (17) give the normalized thermal efficiency as

$$\frac{\eta_{th}}{\eta_{\max}} = \frac{\frac{R_L}{R}\left(\sqrt{1+\frac{ZT_c}{2}\left(1+\left(\frac{T_c}{T_h}\right)^{-1}\right)}+\frac{T_c}{T_h}\right)}{\left[\left(\frac{R_L}{R}+1\right)-\frac{1}{2}\left(1-\frac{T_c}{T_h}\right)+\frac{\left(\frac{R_L}{R}+1\right)^2\frac{T_c}{T_h}}{ZT_c}\right]\left(\sqrt{1+\frac{ZT_c}{2}\left(1+\left(\frac{T_c}{T_h}\right)^{-1}\right)}-1\right)} \quad (23)$$

Note that the above normalized values in Equations (20) – (22) are a function only of R_L/R, while Equation (23) is a function of three parameters, which are T_c/T_h, R_L/R and ZT_c. Also, note that the present analysis is on the basis of T_c.

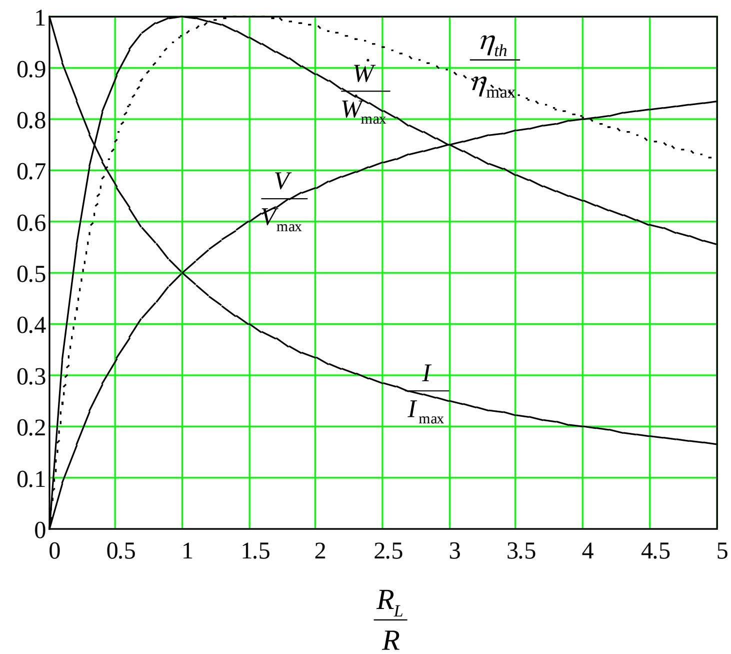


Figure 3. Normalized chart I, where T_c/T_h = 0.7 and ZT_c = 1 are used.

It is first noted, as shown in Figures 3 and 4, that the maximum power output and the maximum conversion efficiency appear close each other with respect to R_L/R. η_{mp} occurs at $R_L/R = 1$, while $\eta_{\max}$ occurs approximately at $R_L/R = 1.5$. The maximum conversion efficiency $\eta_{\max}$ is presented in Figure 5 as a function of both the dimensionless figure of merit (ZT_c) and T_c/T_h. Considering the conventional combustion process (where the thermal efficiency is about 30%) where the high and low junction temperatures would be typically at 1000 K and 400 K, which leads to T_c/T_h = 0.4. Therefore, in order to compete the conventional way of the thermal conversion (30%), the thermoelectric material should be at least ZT_c = 3, which has been the goal. Much development is needed when considering the current technology of thermoelectric material of $ZT_c = 1$. However, there is a strong potential that the nanotechnology would provide a solution toward $ZT_c = 3$.

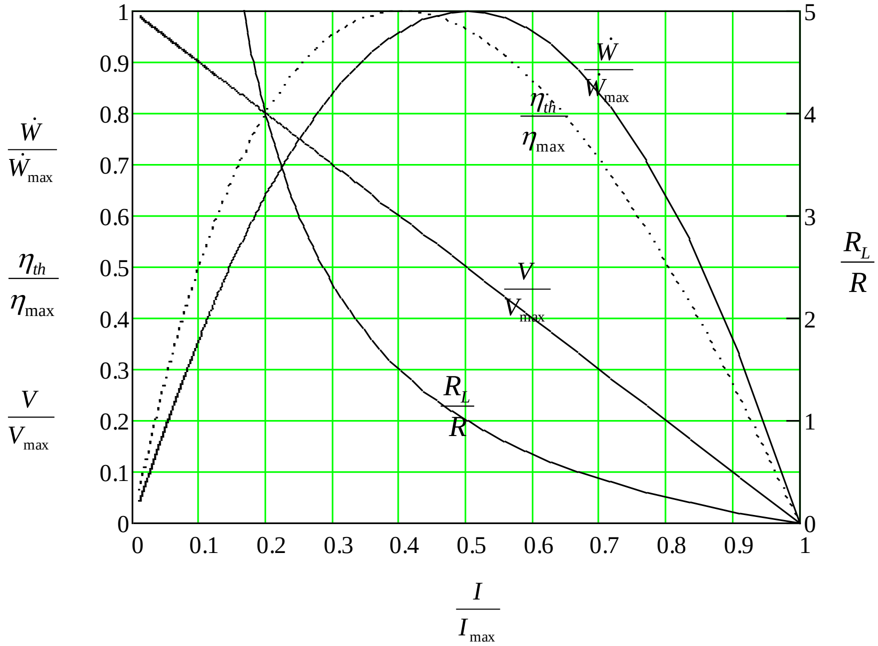


Figure 4. Normalized chart II, where $T_c/T_h = 0.7$ and $ZT_c = 1$ are used.

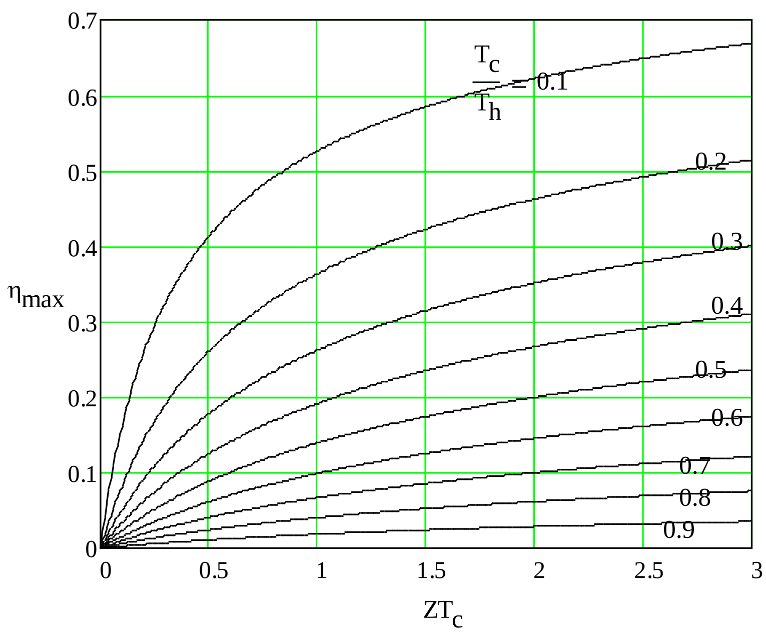
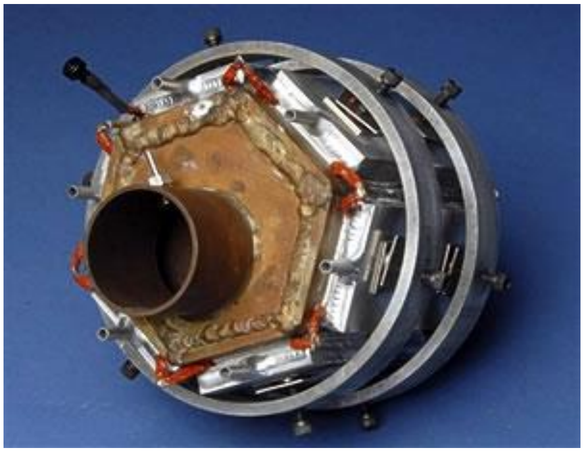


Figure 5. The maximum conversion efficiency versus ZT_c as a function of thte temperature ratio T_c/T_h.

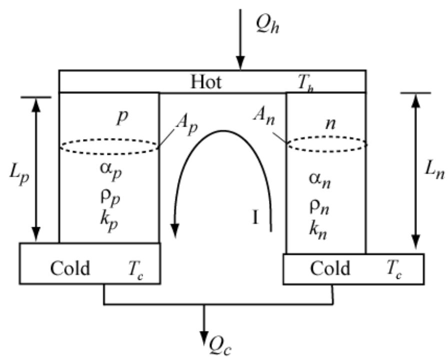
Example E-1

We want to recover waste heat from the exhaust gas of a car using thermoelectric generator (TEG) modules as shown in Figure E-1a. An array of N = 24 TEG modules is installed on the exhaust of the car. Each module has n = 98 thermocouples that consist of p-type and n-type thermoelements. Exhaust gases flow through the TEG device, wherein one side of the modules experiences the exhaust gases while the other side of the modules experiences coolant flows. These cause the hot and cold junction temperatures of the modules to be at 230 °C and 50 °C, respectively. To maintain the junction temperatures, the significant amount of heat should be absorbed at the hot junction and liberated at the cold junction, which usually achieved by heat sinks. The material properties of bismuth telluride for the *p*-type and *n*-type thermoelements are assumed to be similar as $\alpha_p = -\alpha_n$ = 168 µV/K, $\rho_p = \rho_n = 1.56 \times 10^{-3}$ Ωcm, and $k_p = k_n = 1.18 \times 10^{-2}$ W/cmK. The cross-sectional area and leg length of the thermoelement are $A_n = A_p = 12$ mm^2 and $L_n = L_p$ = 4.6 mm, respectively, which are shown in Figure E-1b.

(a) Per one TEG module, compute the electric current, the voltage, the maximum power output, and the maximum power efficiency.
(b) For the whole TEG device, compute the maximum power output, the maximum power efficiency, the maximum conversion efficiency and the total heat absorbed at the hot junction.



(a)



(b)

Figure E-1 (a) TEG device, (b) thermocouple.

Solution:

Material properties: $\alpha = \alpha_p - \alpha_n = 336 \times 10^{-6}$ V/K, $\rho = \rho_p + \rho_n = 3.12 \times 10^{-5}$ Ωm, and $k = k_p + k_n = 2.36$ W/mK

The figure of merit is

$$Z = \frac{\alpha^2}{\rho k} = \frac{\left(336 \times 10^{-6} V/K\right)^2}{\left(3.12 \times 10^{-5} \Omega m\right)\left(2.36 W/mK\right)} = 1.533 \times 10^{-3} K^{-1}$$

and

$$ZT_c = \left(1.533\times10^{-3}K^{-1}\right)\left(323K\right) = 0.495$$

For the maximum power output, we use the condition of $R_L/R = 1$. The internal resistance R is

$$R = \frac{\rho L}{A} = \frac{\left(3.12\times10^{-5}\Omega m\right)\left(4.6\times10^{-3}m\right)}{12\times10^{-6}m^2} = 0.012\Omega$$

(a) For one TEG module:

Using Equation (9), the electric current per module is

$$I = \frac{\alpha\left(T_h - T_c\right)}{R_L + R} = \frac{\left(336\times10^{-6}V/K\right)\left[(230+273)K - (50+273)K\right]}{0.012\Omega + 0.012\Omega} = 2.528A$$

Using Equation (10), the voltage per module is

$$V_n = \frac{n\alpha\left(T_h - T_c\right)}{\frac{R_L}{R}+1}\left(\frac{R_L}{R}\right) = \frac{98\times\left(336\times10^{-6}V/K\right)\left[(230+273)K - (50+273)K\right]}{1+1} = 2.964V$$

Using Equation (11), the maximum power output is

$$\dot{W}_n = \frac{n\alpha^2\left(T_h - T_c\right)^2}{R}\frac{\frac{R_L}{R}}{\left(1+\frac{R_L}{R}\right)^2} = \frac{98\times\left(336\times10^{-6}V/K\right)^2\left[503K - 323K\right]^2}{0.012\Omega\times2^2} = 7.493W$$

Using Equation (19), the maximum power efficiency is

$$\eta_{mp} = \frac{1-\frac{T_c}{T_h}}{2-\frac{1}{2}\left(1-\frac{T_c}{T_h}\right)+\frac{4\frac{T_c}{T_h}}{ZT_c}} = \frac{1-\frac{323K}{503K}}{2-\frac{1}{2}\left(1-\frac{323K}{503K}\right)+\frac{4\frac{323K}{503K}}{0.495}} = 0.051$$

(b) For the whole TEG device:

The maximum power output is

$$\dot{W}_n = 24 \times 7.493W = 179.8W$$

The maximum power efficiency is same as the one for the module, so
$\eta_{mp} = 0.051$

Using Equation (17), the maximum conversion efficiency is

$$Z\overline{T} = Z\left(\frac{T_c + T_h}{2}\right) = \left(1.533 \times 10^{-3} K^{-1}\right)\left(\frac{323K + 503K}{2}\right) = 0.633$$

$$\eta_{\max} = \left(1 - \frac{T_c}{T_h}\right)\frac{\sqrt{1 + Z\overline{T}} - 1}{\sqrt{1 + Z\overline{T}} + \frac{T_c}{T_h}} = \left(1 - \frac{323K}{503K}\right)\frac{\sqrt{1 + 0.633} - 1}{\sqrt{1 + 0.633} + \frac{323K}{503K}} = 0.052$$

The total heat absorbed is

$$\dot{Q}_h = \frac{\dot{W}_n}{\eta_{mp}} = \frac{179.8W}{0.051} = 3{,}525W$$

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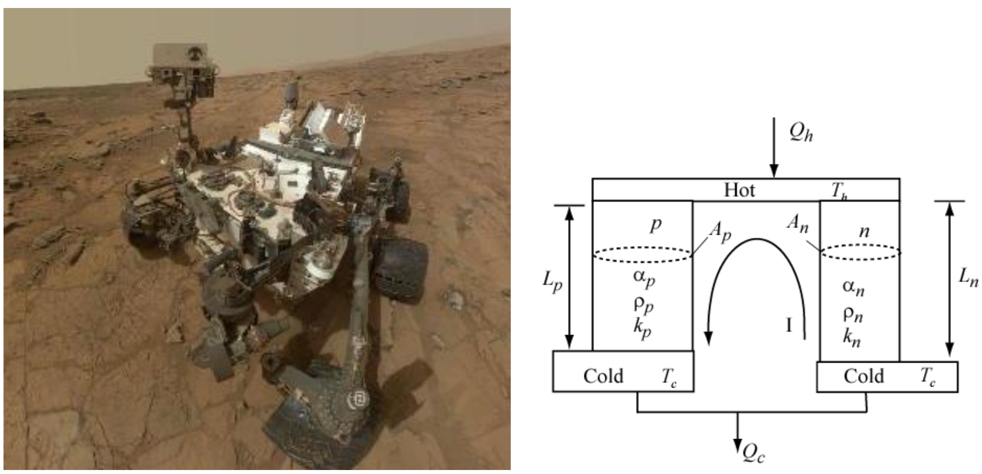
Problem P-1

A NASA's Curiosity rover is working (February, 2013) on the Mars surface to collect a sample of bedrock that might offer evidence of a long-gone wet environment, as shown in Figure P-1a. In order to provide the electric power for the work, radioisotope thermoelectric generator (RTG) wherein Plutonium fuel pellets provide thermal energy is used. The p-type and n-type thermoelements are assumed to be similar having the dimensions as the cross-sectional area A = 0.196 cm^2 and the leg length L = 1 cm. The thermoelectric material used is lead telluride (PbTe) having $\alpha_p = -\alpha_n = 187$ μV/K, $\rho_p = \rho_n = 1.64 \times 10^{-3}$ Ωcm, and $k_p = k_n = 1.46 \times 10^{-2}$ W/cmK. The hot and cold junction temperatures are at 815 K and 483 K, respectively. If the power output of 123 W is required to fulfill the work, estimate the number of thermocouples, the maximum power efficiency and the rate of heat liberated at the cold junction of the RTG.



(a) (b)

Figure P-1. (a) Curiosity rover on Mars, (b) p-type and n-type thermoelements.